A biological arrow of time and computational undecidability

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PAPER

Biological arrow of time: emergence of tangled information hierarchies and self-modelling dynamics

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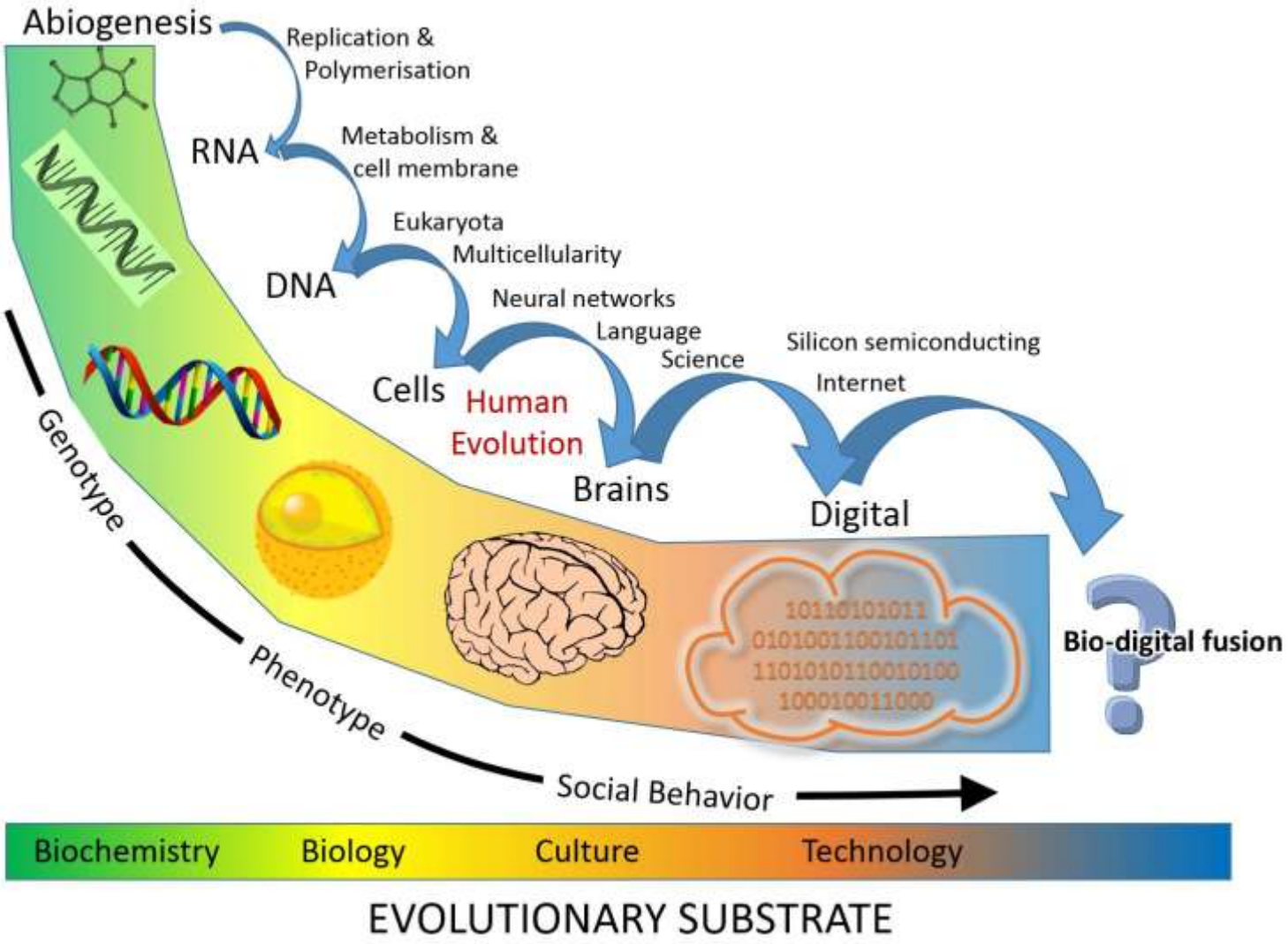
Outline

- Major evolutionary transitions and “codes of life”
- Coding thresholds and “interaction at a distance”
- Self-reference and incomputability
- Turing oracle machines and Turing jumps
- Meta-simulation and biological arrow of time

Major Evolutionary Transitions



based on: Gillings, M. R., Hilbert, M., & Kemp, D. J. (2016). Information in the Biosphere: Biological and Digital Worlds. *Trends in Ecology & Evolution*, 31(3), 180–189. http://escholarship.org/uc/item/38f4b791

Wikipedia

Major Evolutionary Transitions & "codes of life"

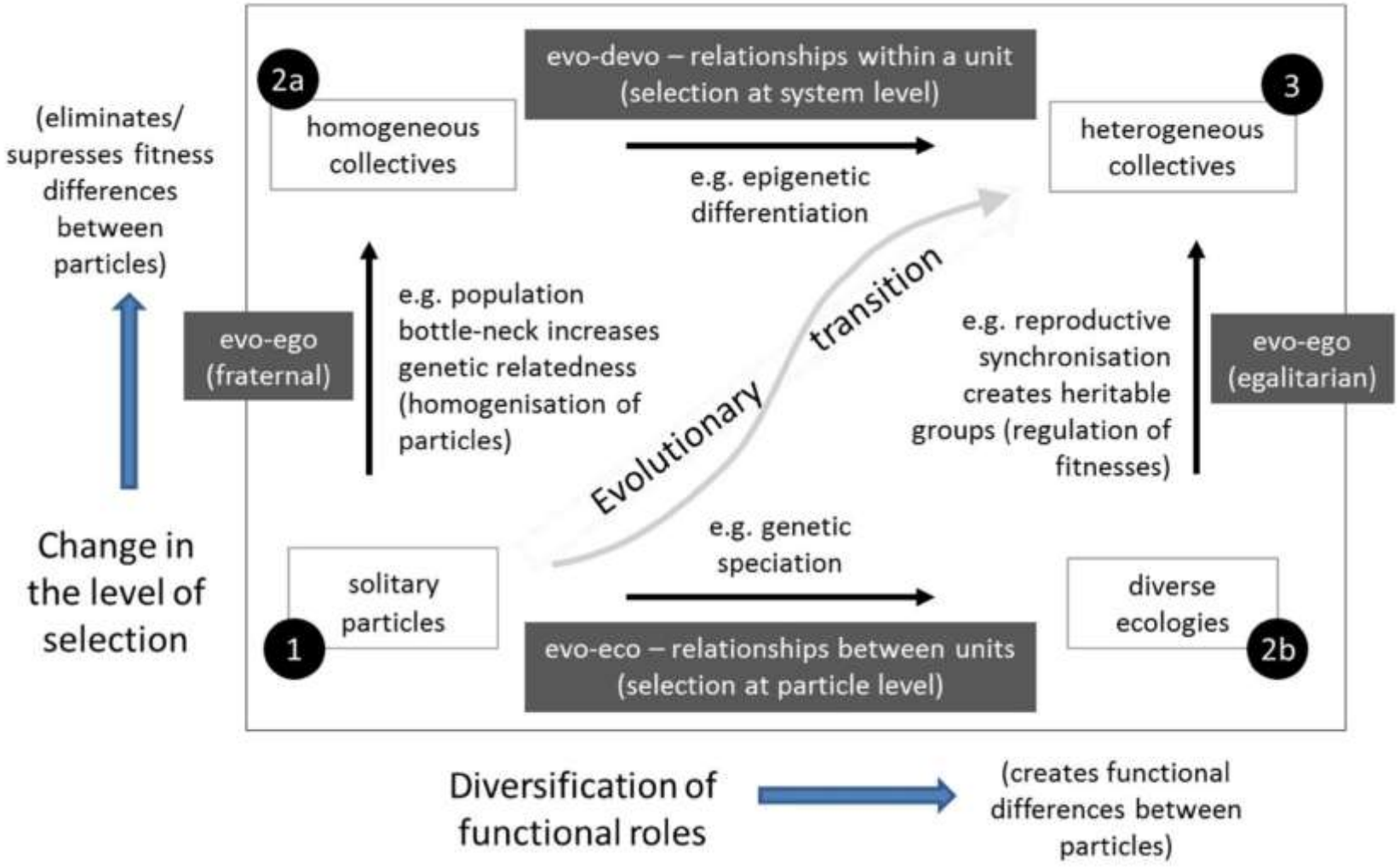
TABLE 1 The major transitions[1]

Replicating molecules to populations of molecules in compartments
Unlinked replicators to chromosomes
RNA as gene and enzyme to DNA and protein (genetic code)
Prokaryotes to eukaryotes
Asexual clones to sexual populations
Protists to animals, plants and fungi (cell differentiation)
Solitary individuals to colonies (non-reproductive castes)
Primate societies to human societies (language)

…the emergence of novel inheritance systems, involving new modes of storing, transmitting, and processing information — new "codes of life"

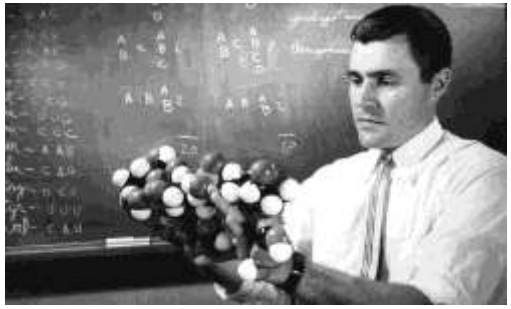
Szathmáry E., Maynard Smith J. The major evolutionary transitions, *Nature*, 374, 227-232, 1995.
Kun A. The major evolutionary transitions and codes of life, *BioSystems*, 210, 104548, 2021.

Evolutionary transitions (Watson et al.)



Watson R. A. et al. Evolutionary connectionism: algorithmic principles underlying the evolution of biological organisation in evo-devo, evo-eco and evolutionary transitions, *Evolutionary Biology*, 43, 553 - 581, 2016.

Novelty generation and “coding thresholds”



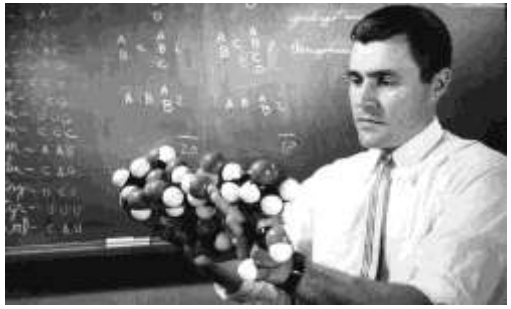
“...the overwhelming amount of novelty needed to bring modern cells into existence ...is the central and most challenging question...”

“Somewhere along the line there had to have occurred a saltation that we could call the “coding threshold,” where the capacity to represent nucleic acid sequence symbolically in terms of a (colinear) amino acid sequence developed, a development that would generate a truly enormous new, totally unique evolutionary phase space.”

Woese, C.R., A new biology for a new century, *Microbiology and Molecular Biology Reviews*, 68(2):173-186, 2004.

Novelty generation and “coding thresholds”

“… Another good critical-point candidate is the advent of (eucaryotic) multicellularity. Here too the saltation is accompanied by a qualitatively new world of possibilities.”



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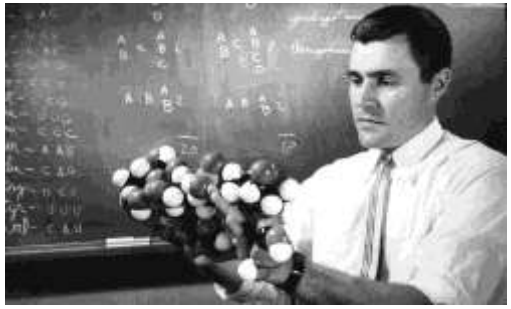
Novelty generation and "coding thresholds"



"… Another good critical-point candidate is the advent of (eucaryotic) multicellularity. Here too the saltation is accompanied by a qualitatively new world of possibilities."

"…It is immediately apparent that one of them is the development of language(s). Human language is a development that has set Homo sapiens worlds apart from its otherwise very close primate relatives, adding new dimensions to the phase space within which human evolution occurs."

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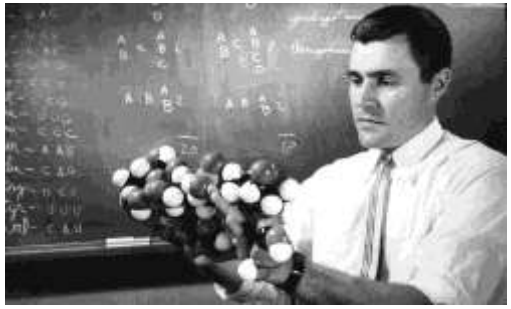

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"A common thread that links language and multicellularity is communication (interaction at a distance). In each case a complex, sophisticated network of interactions forms the medium within which the new level of organization (entities) comes into existence."

Woese, C.R., A new biology for a new century, *Microbiology and Molecular Biology Reviews*, 68(2):173-186, 2004.

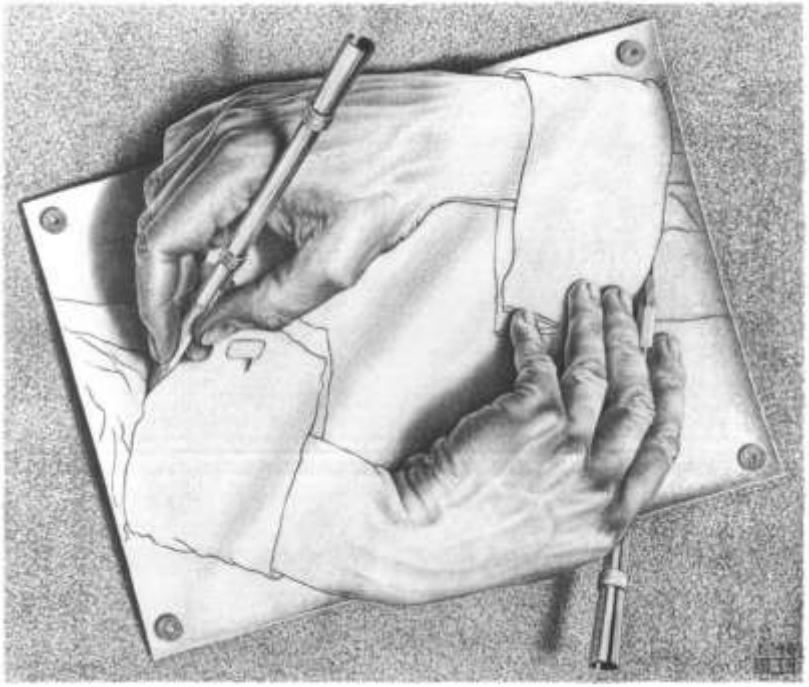
Open ended evolution

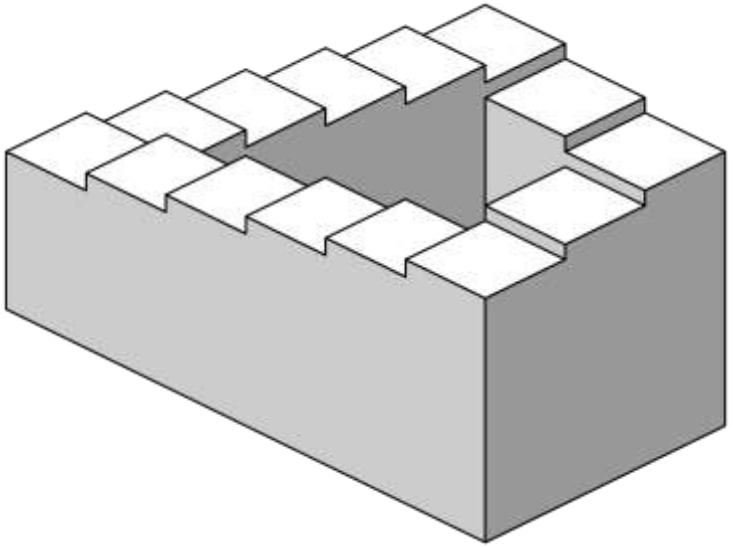
"These rules themselves need to evolve, but how? We need an additional set of rules describing the evolution of the original rules.

But this upper level of rules itself needs to evolve. Thus, we end up with an infinite hierarchy, an inevitable reflection of the fact that the dynamic we are seeking is inherently self-referential."

Goldenfeld N., Woese C. Life is physics: evolution as a collective phenomenon far from equilibrium, *Annual Review of Condensed Matter Physics*, 2, 375-399, 2011.

Tangled hierarchies and strange loops





Oscar Reutersvärd **(1937)** /
Lionel Penrose & Roger Penrose **(1958)**

Drawing Hands:

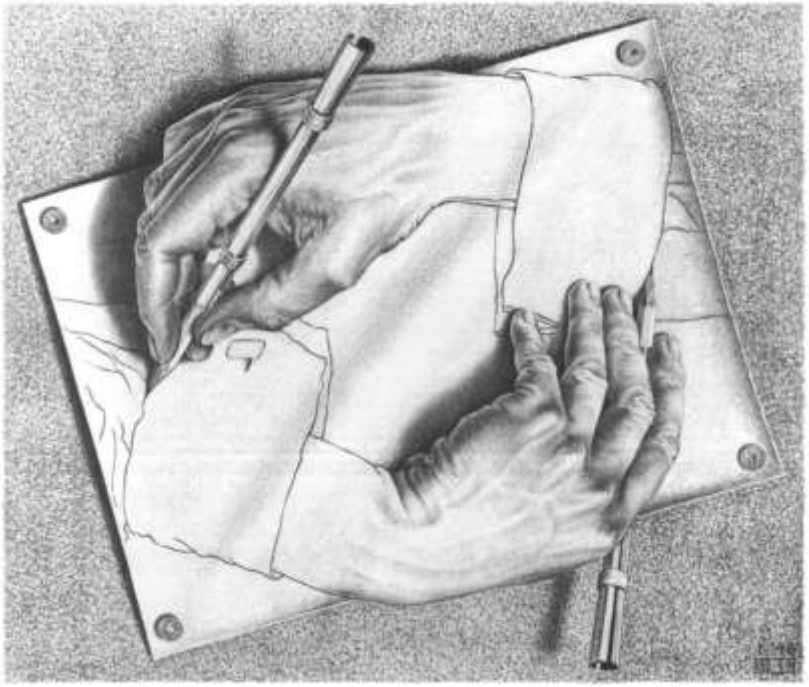
the product, the hands, are undertaking the operation – the drawing of the hands

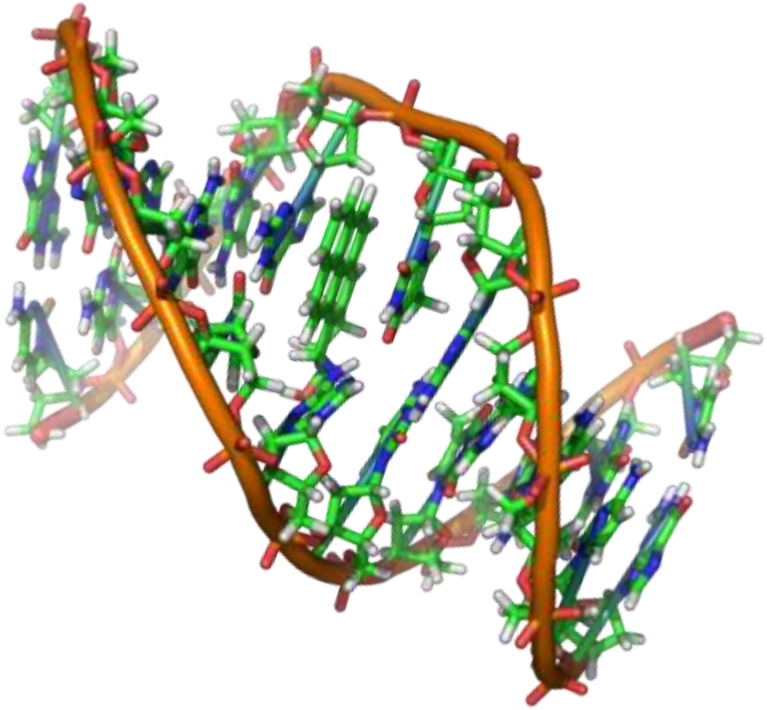
Maurits Cornelis Escher **(1948)**

"an interaction between levels in which the top level reaches back down towards the bottom level and influences it, while at the same time being itself determined by the bottom level"

— Hofstadter **(1989)**

Self and a functional description of “self”





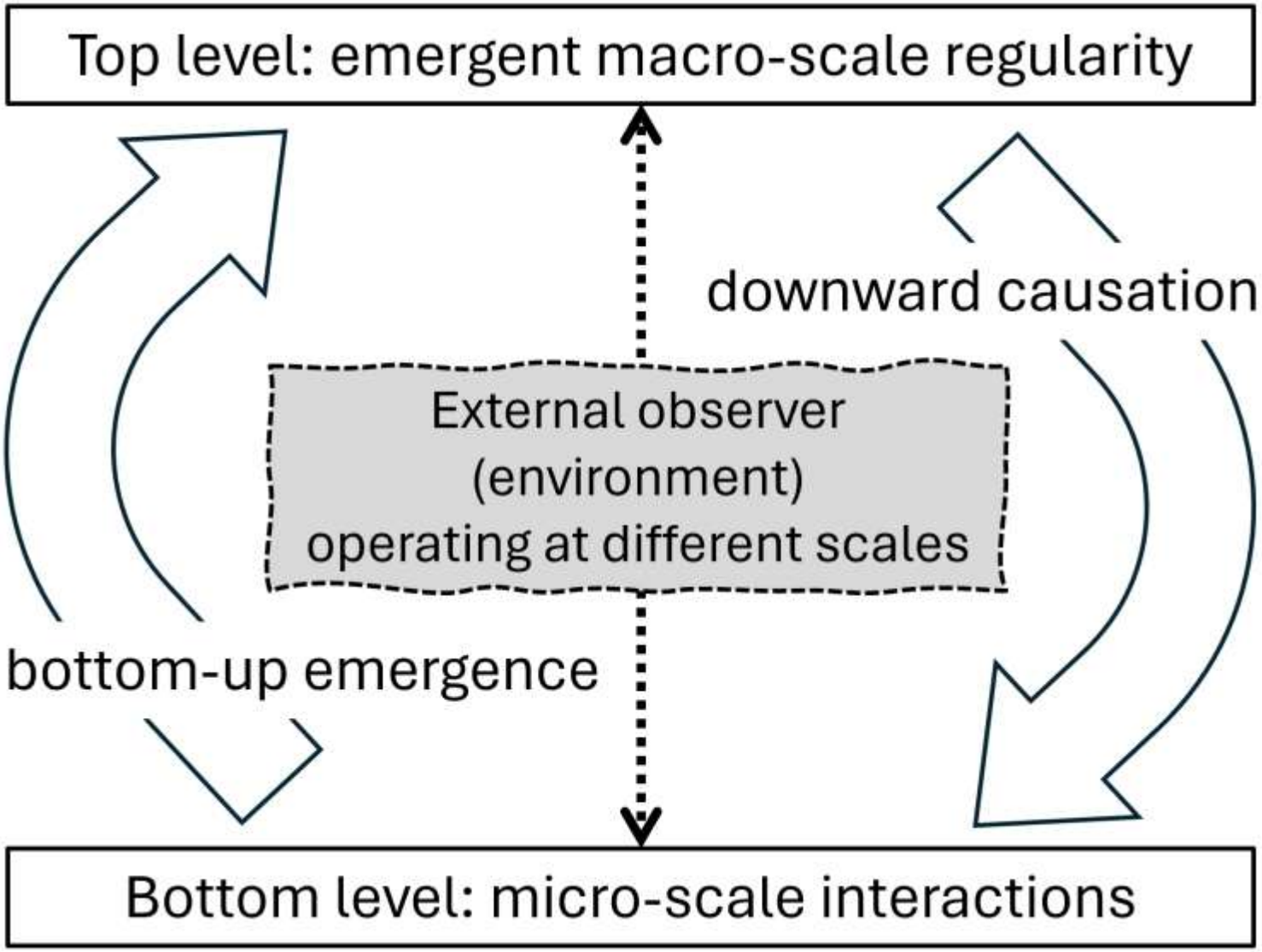
Drawing Hands:
the product, the hands, are undertaking the operation – the drawing of the hands

DNA: genetic instructions (*sequence*) used in development and functioning of a living organism (*function and structure*) – needed to construct other components of cells, and copy itself

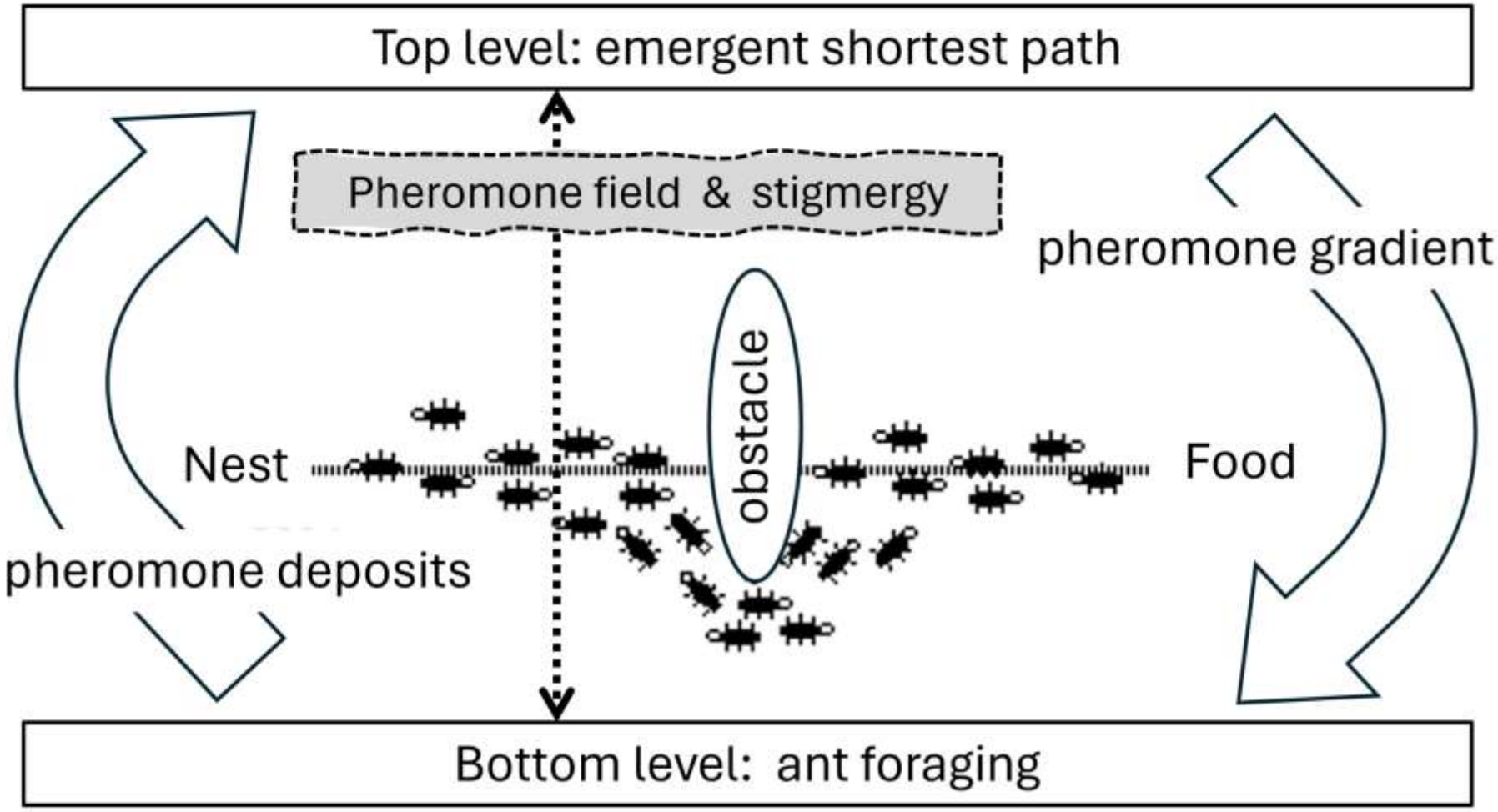
M. C. Escher, Drawing Hands, Wikipedia

Wikipedia

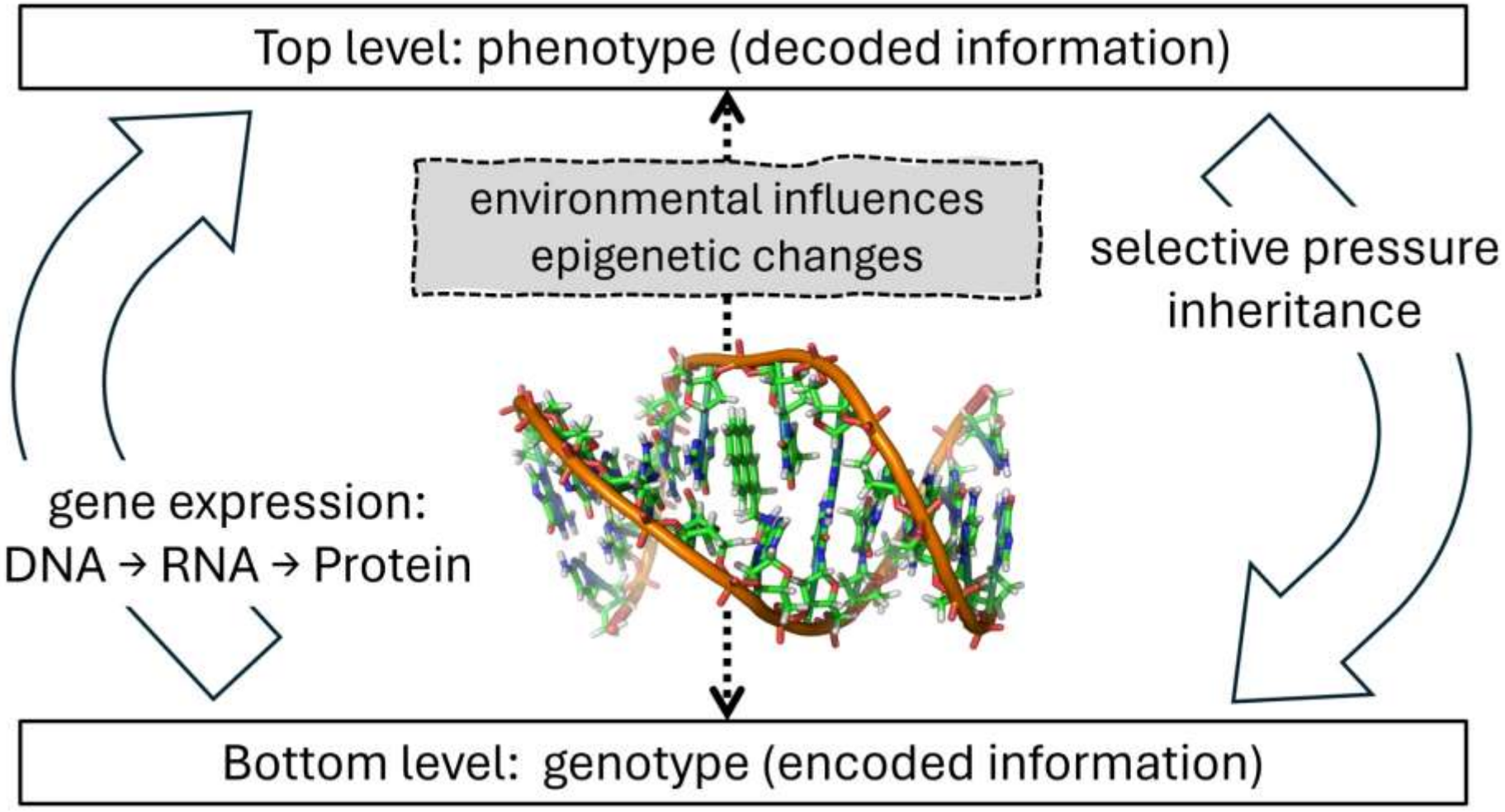
Tangled hierarchy



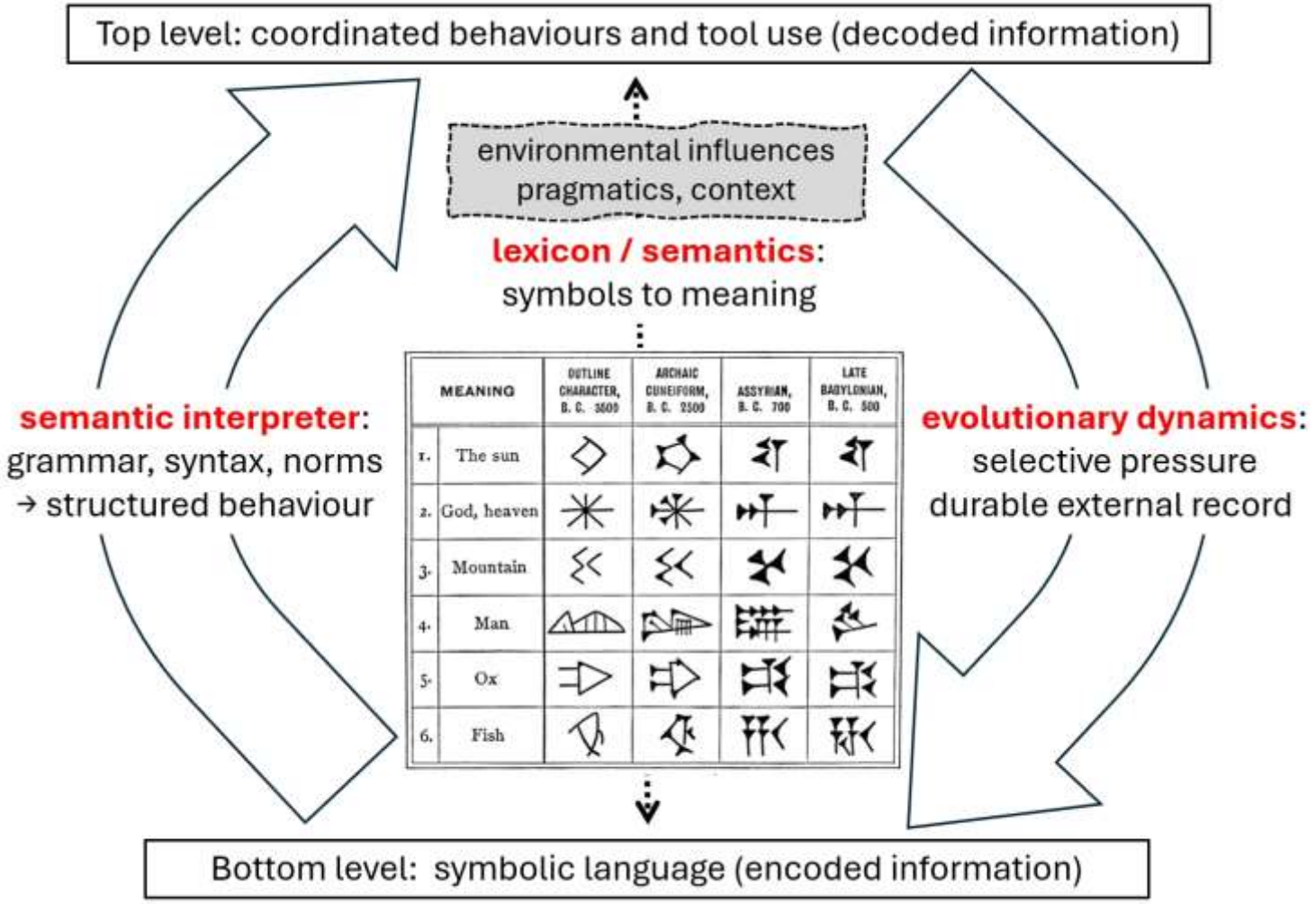
Tangled hierarchy: type I



Tangled hierarchy with self-modelling: type II



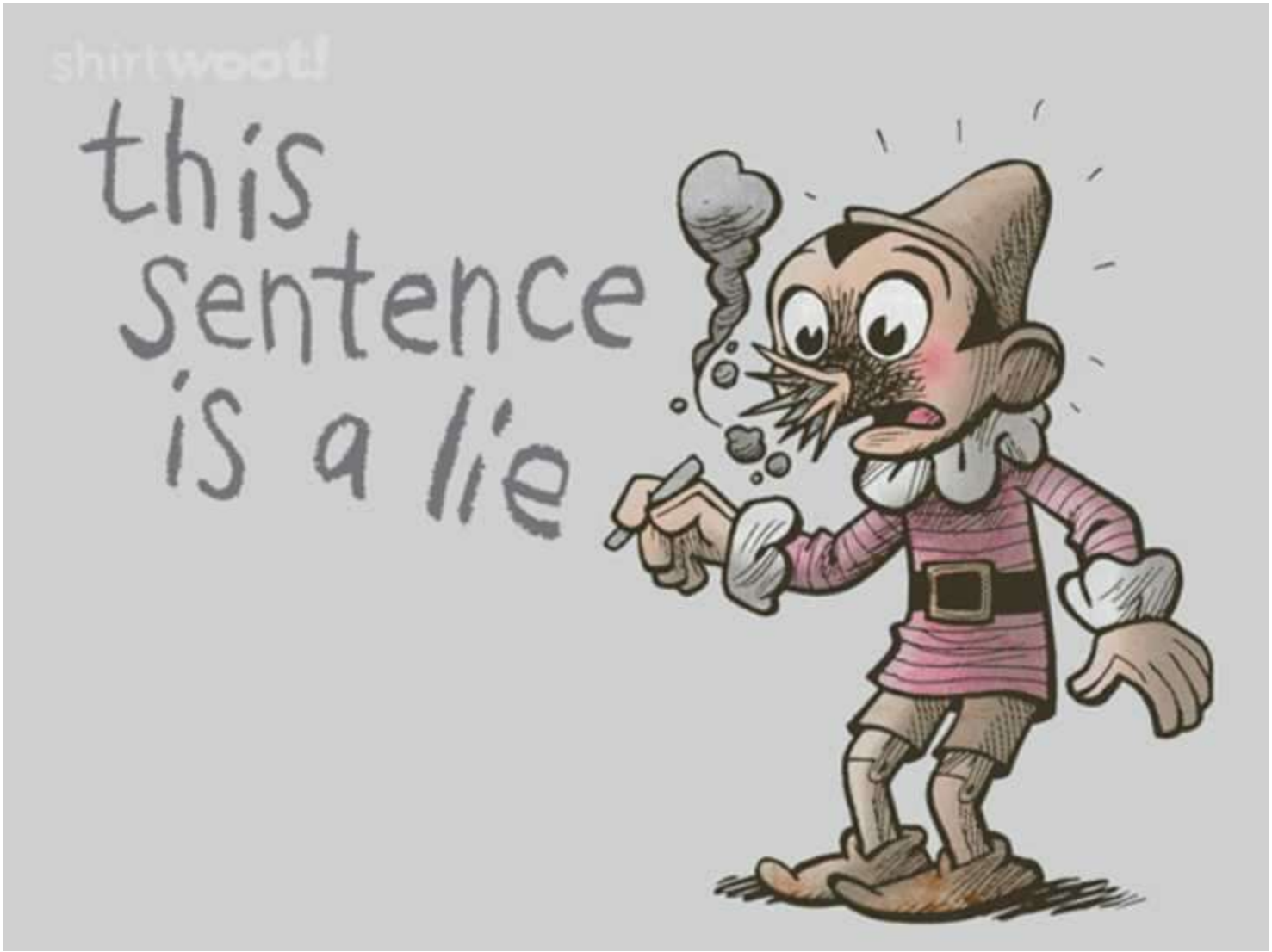
Tangled hierarchy with self-modelling: type II



	MEANING	OUTLINE CHARACTER, B. C. 3500	ARCHAIC CUNEIFORM, B. C. 2500	ASSYRIAN, B. C. 700	LATE BABYLONIAN, B. C. 500
1.	The sun				
2.	God, heaven				
3.	Mountain				
4.	Man				
5.	Ox				
6.	Fish				

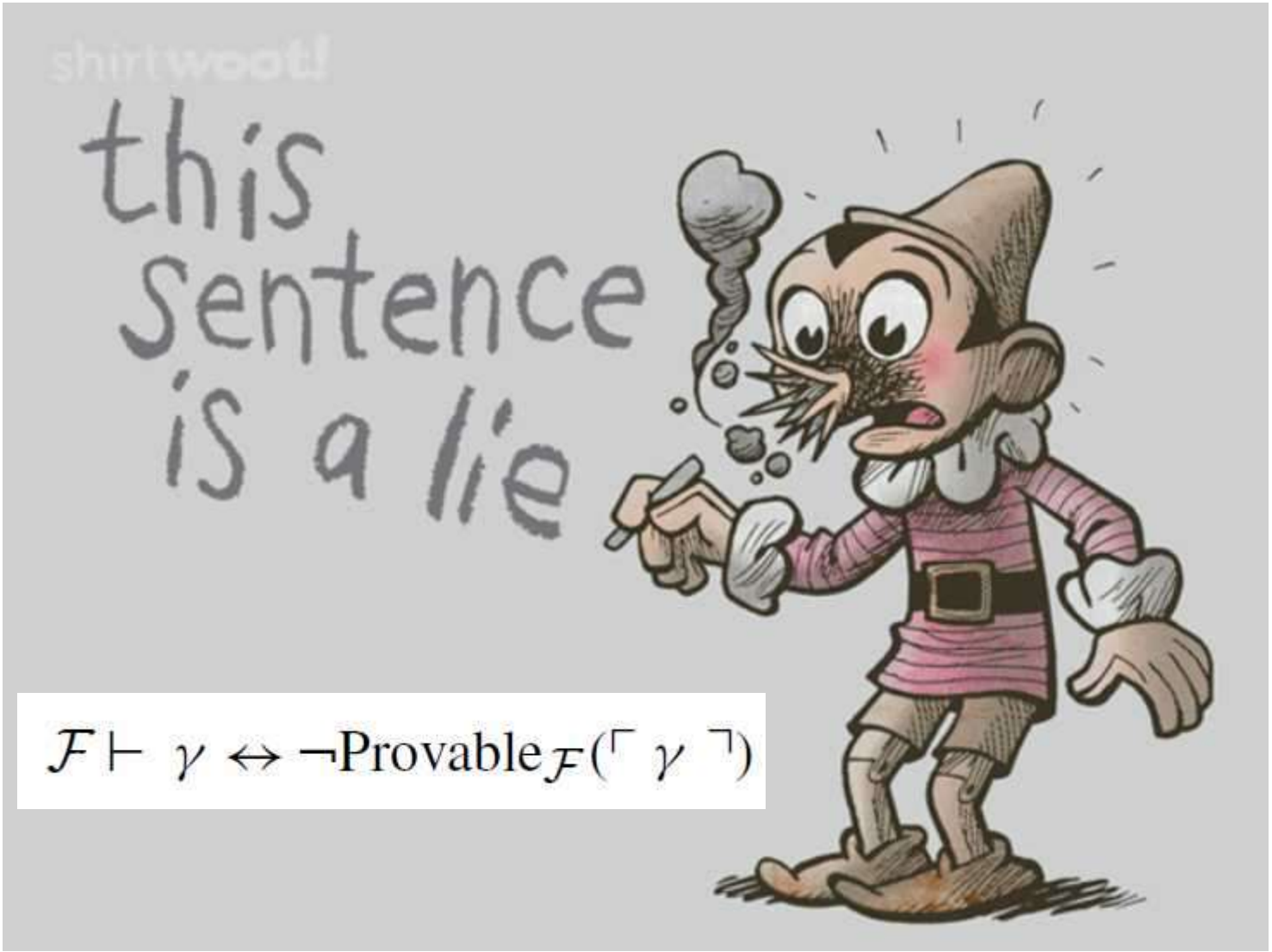
M. Prokopenko, et al., Algorithmic bottlenecks in evolution: Genetic code, symbolic language, and the Great Filter hypothesis, *Physics of Life Reviews*, 59: 96-119, 2026.

Self-reference and the Liar paradox



https://shirt.woot.com/offers/liars-paradox

The Liar paradox and Gödel sentence



https://shirt.woot.com/offers/liars-paradox

Gödel: incompleteness theorems

First Incompleteness *Theorem*: *"Any consistent formal system F within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of F which can neither be proved nor disproved in F."* **(1931)**

Second Incompleteness Theorem: *"For any consistent system F within which a certain amount of elementary arithmetic can be carried out, the consistency of F cannot be proved in F itself."* **(1931)**

Wikipedia

Incomputability and diagonalisation arguments

referents

expressions

	$\ulcorner E_0 \urcorner$	$\ulcorner E_1 \urcorner$	$\ulcorner E_2 \urcorner$	$\cdots$	$\ulcorner E_k \urcorner$	$\cdots$
E_0	P_{00}	P_{01}	P_{02}	$\cdots$	P_{0k}	$\cdots$
E_1	P_{10}	P_{11}	P_{12}	$\cdots$	P_{1k}	$\cdots$
E_2	P_{20}	P_{21}	P_{22}	$\cdots$	P_{2k}	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
E_k	P_{k0}	P_{k1}	P_{k2}	$\cdots$	P_{kk}	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

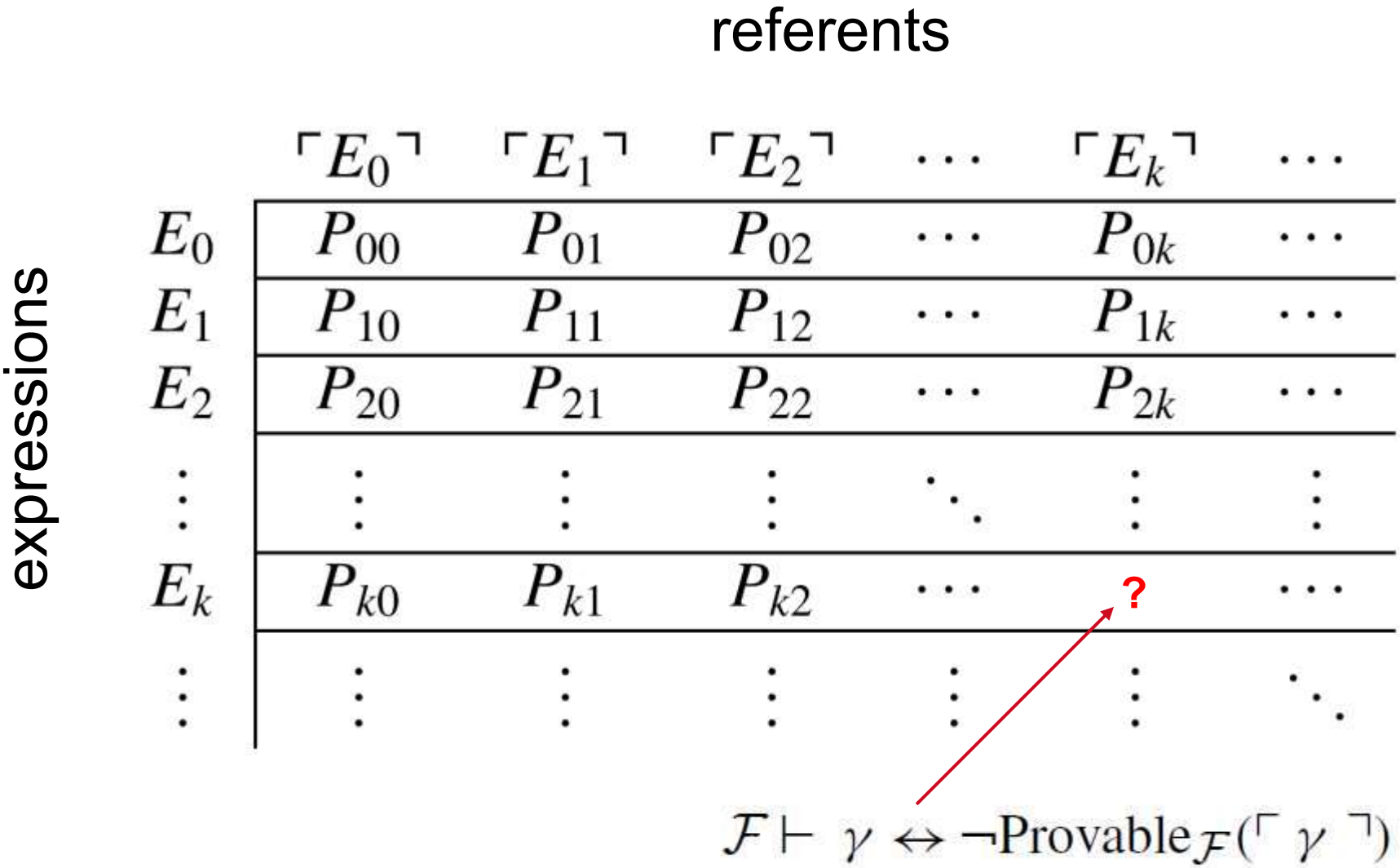
Incomputability and diagonalisation arguments

referents

expressions

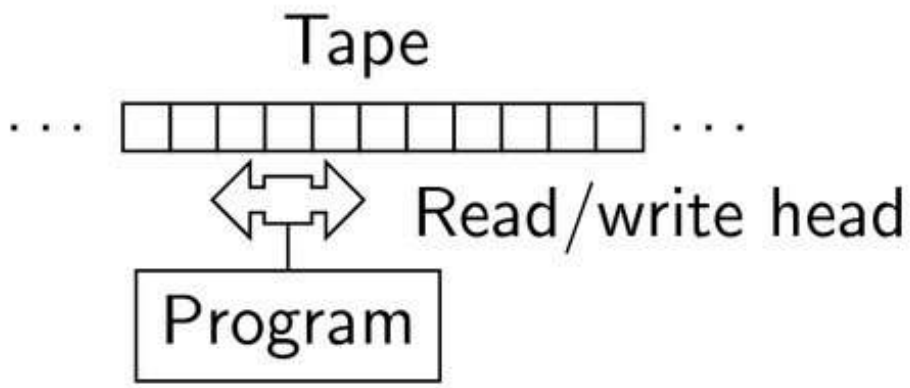
	$\ulcorner E_0 \urcorner$	$\ulcorner E_1 \urcorner$	$\ulcorner E_2 \urcorner$	$\cdots$	$\ulcorner E_k \urcorner$	$\cdots$
E_0	P_{00}	P_{01}	P_{02}	$\cdots$	P_{0k}	$\cdots$
E_1	P_{10}	P_{11}	P_{12}	$\cdots$	P_{1k}	$\cdots$
E_2	P_{20}	P_{21}	P_{22}	$\cdots$	P_{2k}	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
E_k	P_{k0}	P_{k1}	P_{k2}	$\cdots$	?	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

Incomputability and diagonalisation arguments



Turing Machine

Turing machines
alphabets: input Σ and tape Γ
tape strings in Σ^*
admissible syntax, given Γ, e.g., blank symbol
recognizable tape pattern in Σ^*, given Γ
infinite tape



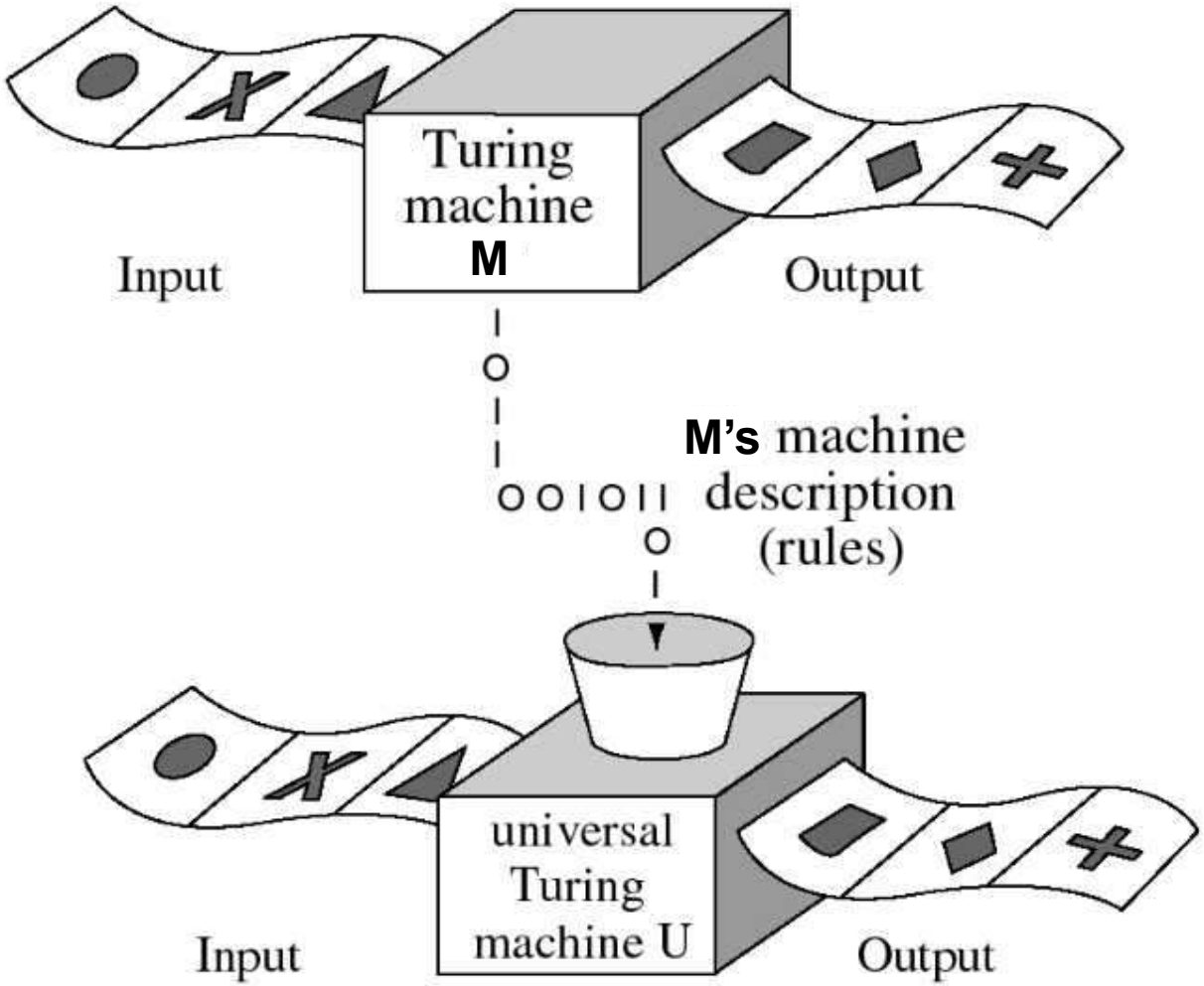
This "universal computing machine" would be capable of performing any conceivable mathematical computation if it were representable as an algorithm.

BUT: It is not possible to decide algorithmically whether a Turing machine will ever halt **(1937).**

— Avi Wigderson **(2019)**

Ludger Humbert / TEXample / CC BY 2.5; http://www.texample.net/tikz/examples/turing-machine/

Universal Turing Machine: can simulate any other machine



Glenn D. Blank et al., 2003

http://www.cse.lehigh.edu/~glennb/um/1intro.pdf

Universal Decider Turing Machine: does not exist!

$$P([M,w]) = \begin{cases} accept & \text{if } M \text{ accepts } w \\ reject & \text{if } M \text{ does not accept } w \end{cases}$$

Universal Decider

$$V([M]) = \begin{cases} reject & \text{if } M \text{ accepts } [M] \\ accept & \text{if } M \text{ does not accept } [M] \end{cases}$$

The Liar

$$V([V]) = \begin{cases} reject & \text{if } V \text{ accepts } [V] \\ accept & \text{if } V \text{ does not accept } [V] \end{cases}$$

Self-referential Liar

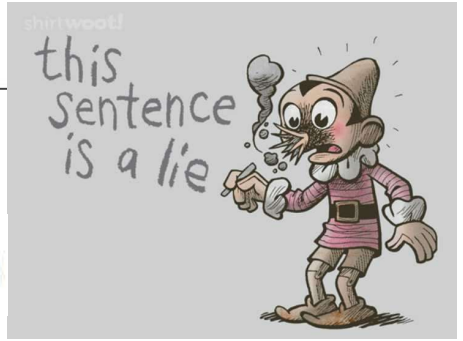
Self-reference and diagonalisation

inverter of universal decider

Table 5
The cell (k, j) is the outcome of running $V = M_k$ (the inverter of P) on $[M_k[M_j]]$. A contradiction occurs at cell (k, k).

	$[M_1]$	$[M_2]$	$[M_3]$	$\cdots$	$[M_k]$	$\cdots$
M_1	accept	reject	accept		accept	
M_2	accept	accept	accept	$\cdots$	reject	$\cdots$
M_3	reject	accept	reject		reject	
$\vdots$		$\vdots$		$\ddots$		
M_k	reject	reject	accept		?	
$\vdots$		$\vdots$				

$$\mathcal{F} \vdash \gamma \leftrightarrow \neg \text{Provable}_{\mathcal{F}}(\ulcorner \gamma \urcorner)$$



M. Prokopenko, et al., Self-referential basis of undecidable dynamics: from the Liar paradox and the halting problem to the edge of chaos. *Physics of Life Reviews*, 31: 134-156, 2019.

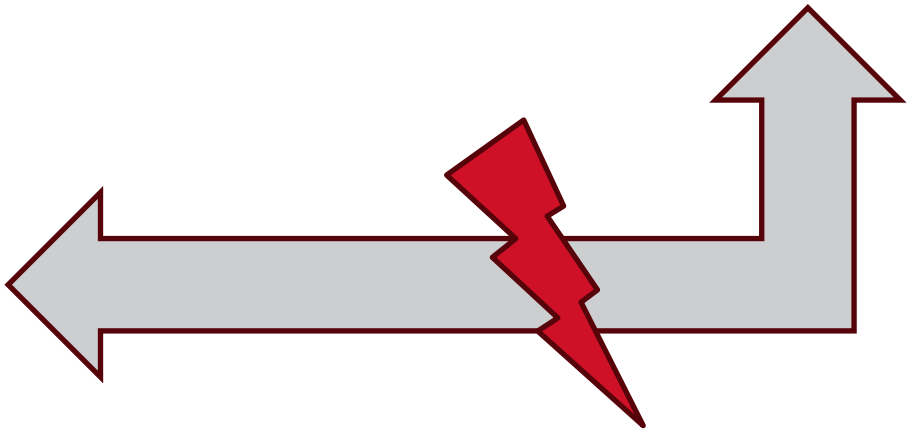
Expression – referent discrepancy

referents

$\ulcorner E_0 \urcorner \quad \ulcorner E_1 \urcorner \quad \ulcorner E_2 \urcorner \quad \ldots \quad \ulcorner E_k \urcorner \quad \ldots$

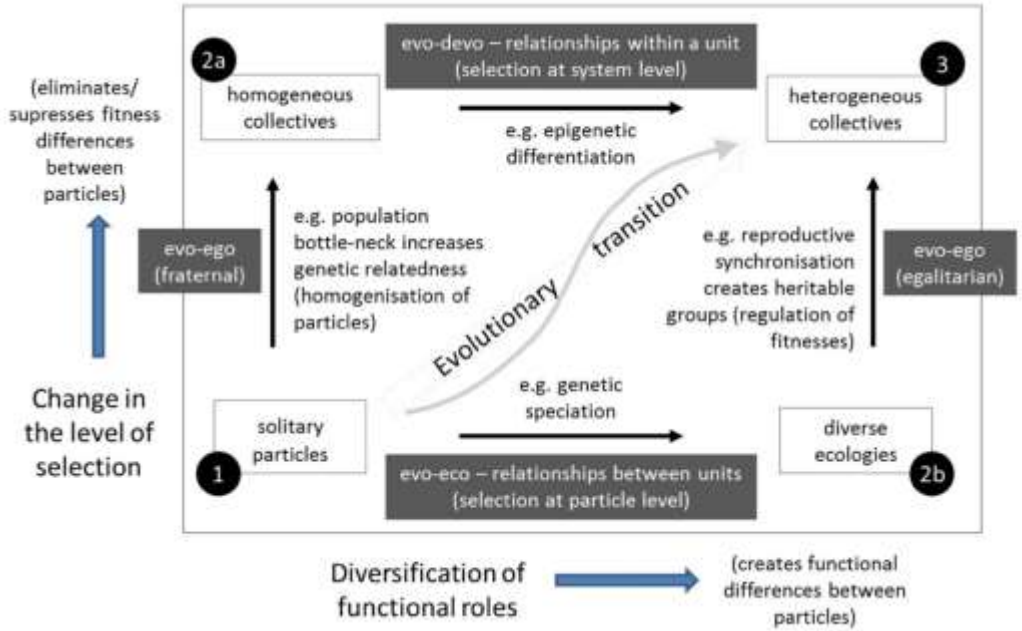
expressions

E_0
E_1
E_2
$\vdots$
E_k
$\vdots$



there are always more expressions (objects) than referents (encodings)

Individual vs group interests



$$y = F(x_1, x_2, \ldots, x_n)$$

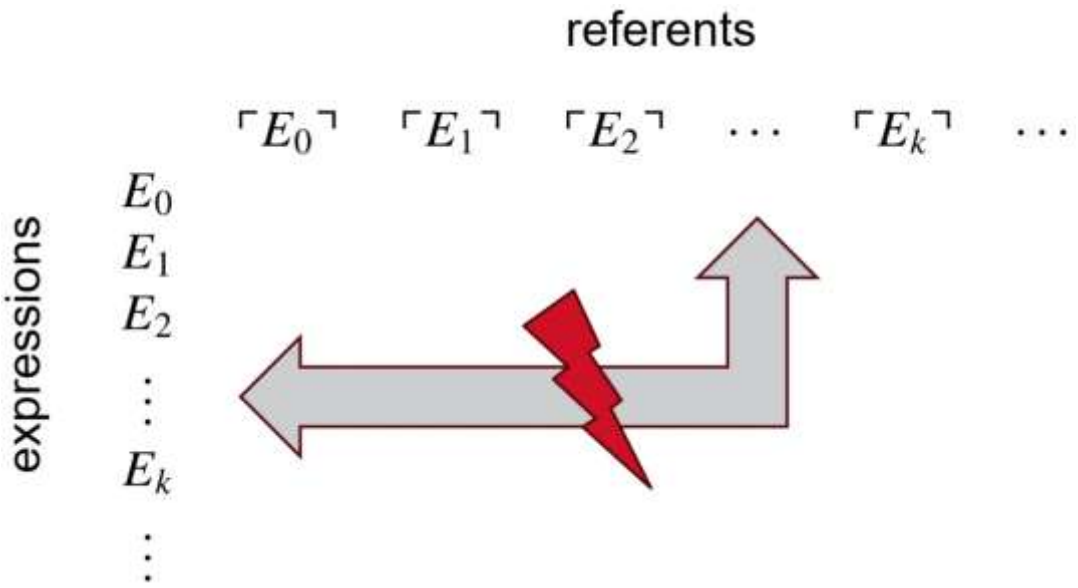
$$\neq$$

$$F = \sum_i f_i(x_i)$$

- when evolved functions are non-linearly separable, the hierarchical structures constitute new individuals
- higher-level individuals enable information integration and collective actions
- when evolved interaction structures between units compute a non-linearly separable function, this makes collective fitness, and hence reproduction, non-decomposable in a formal sense
- not all collective interests are reducible to a (sum of) individual preferences
- evolutionary tension between individual and group interests is an “inconsistency”, directly related to the expression-referent discrepancy

Watson R. A. et al. Evolutionary connectionism: algorithmic principles underlying the evolution of biological organisation in evo-devo, evo-eco and evolutionary transitions, *Evolutionary Biology*, 43, 553 - 581, 2016.

Individual vs group interests: the expression-referent discrepancy



$$y = F(x_1, x_2, \ldots, x_n)$$

$$\neq$$

$$F = \sum_i f_i(x_i)$$

- Determining whether the encoding and realisation are consistent requires solving a fixed-point problem
- The question whether the referents and expressions are consistent becomes undecidable
- Non-separability of collective fitness is a symptom of the referent-expression discrepancy
  - fitness-relevance is a relation among realised elements
  - the realisation contains relations which cannot be localised to individual encoding units
  - ? the larger the distinction between what is encoded locally and what becomes realised globally, the stronger the non-separability

“Jumping out of the system…” (Hofstadter: JOOTSing)

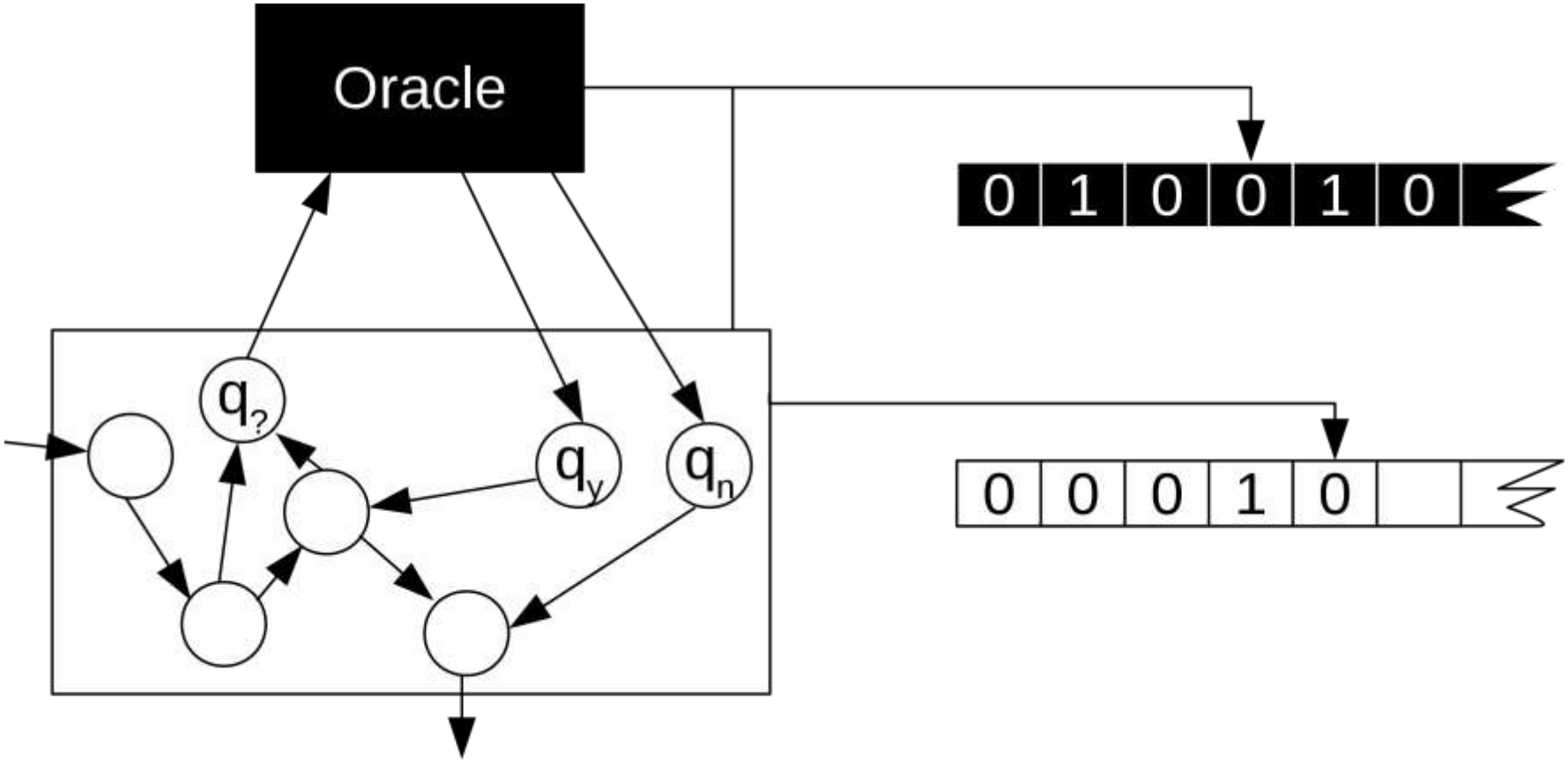
“… we must ask if there exists a larger meta-system, encompassing the computational undecidability, which may play the role of universal decider and inverter… Importantly, an undecidable problem is framed within a given formal system, and once the system is appropriately extended, the problem in point becomes decidable — at the cost of generating other undecidable problems that inevitably arise in the extended system.” [3]

[1] Hofstadter, D. R. *Gödel, Escher, Bach: An eternal golden braid*, 1980.

[2] Soare, R. I., Turing oracle machines, online computing, and three displacements in computability theory, *Annals of Pure and Applied Logic*, 160: 368-399, 2009.

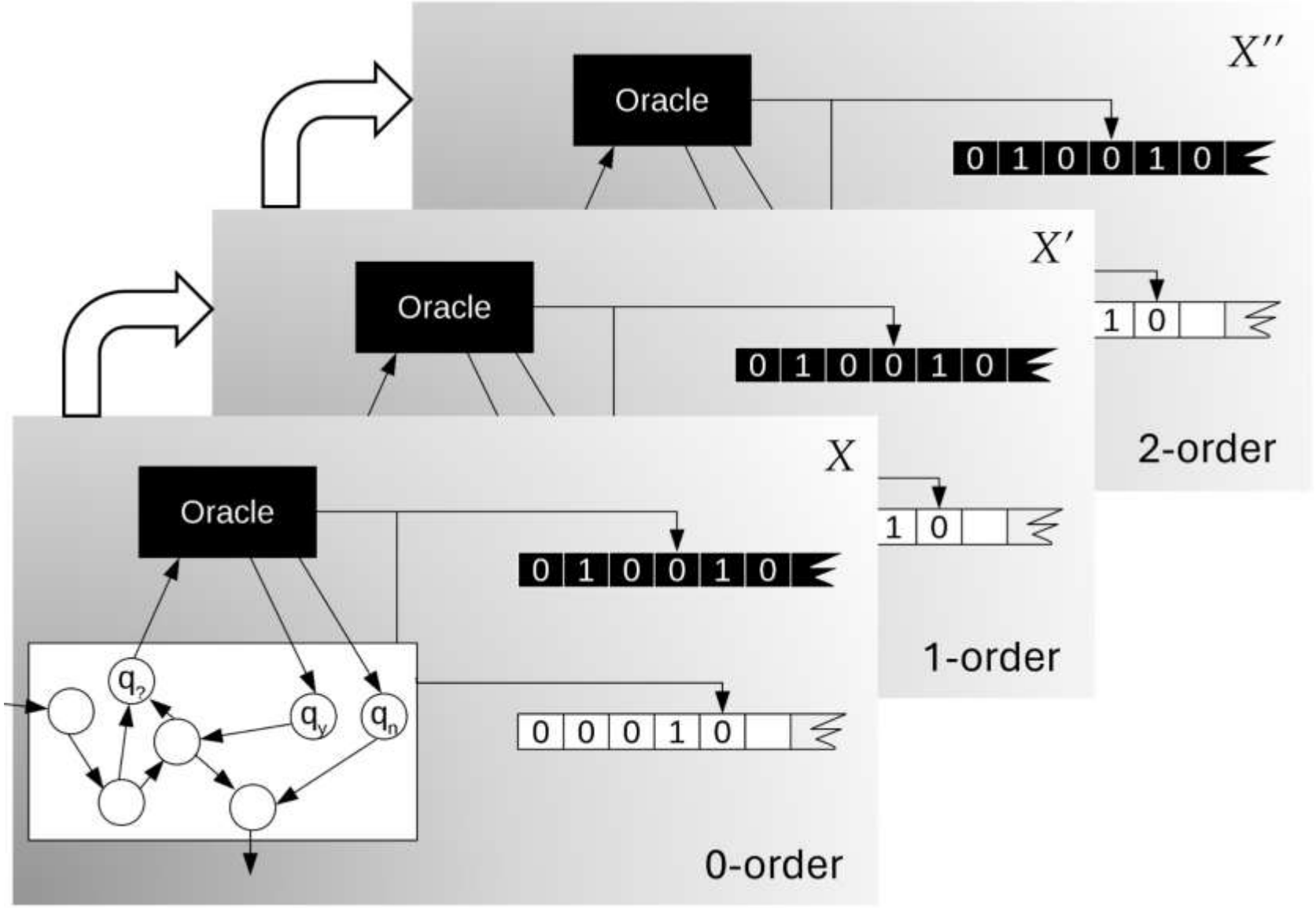
[3] Svahn, A. J., Prokopenko, M., An ansatz for computational undecidability in RNA automata, *Artificial Life*, 29(2): 261-288, 2023.

Oracle machine (Turing)



Wikipedia

α −order Oracle machines and meta-simulation



α −order Oracle machines and Turing jumps

Level	Can solve
0	Ordinary computable problems
$0'$	Halting problem for ordinary machines
$0''$	Halting problem for machines using $0'$
$0'''$	Halting problem for machines using $0''$
...	...

α −order Oracle machines and Turing jumps

Level	Can solve	
0	Ordinary computable problems	
$0'$	Halting problem for ordinary machines	$0' \leftrightarrow \Sigma^0_1 \quad \exists x\, R$
$0''$	Halting problem for machines using $0'$	$0'' \leftrightarrow \Sigma^0_2 \quad \exists x \forall y\, R$
$0'''$	Halting problem for machines using $0''$	
...	...	$0^{(n)} \leftrightarrow \Sigma^0_n$

- Quantifier alternation measures how many layers of dependence are present in an arithmetic definition
- The Turing jump creates the additional computational power needed to decide statements with one more layer of alternation

Turing jumps and non-decomposable fitness

A non-interacting collection of agents may only realise simple, decomposable predicates:

$$F(x_1, \ldots, x_n) = \sum_i f_i(x_i)$$

When interaction structures evolve, the collective can represent nested dependencies, for example:

∃ configuration of some parts such that
∀ perturbations of others
∃ compensatory response
…

- Quantifier alternation measures how many layers of dependence are present in an arithmetic definition (*an aggregate organism*)
- The Turing jump (*transition in individuality*) creates the additional computational power needed to decide statements with one more layer of alternation, but increases the depth and complexity of meta-simulation

Conclusion: biological arrow of time as open-ended meta-simulation

- Emergence of tangled hierarchy without self-modelling (TH type I), with distributed coordination and imprecise replication
- Encapsulation of compressed functional self-description, via encoding (compression) of macro-level information patterns
- Formation of tangled hierarchy with self-modelling (TH type II), with encoding and decoding of compressed patterns
  - Self-reference: expressions becoming referents and vice versa
  - Undecidability, due to the expression-referent discrepancy: there are always more expressions (objects) than referents (encodings)
- Extension of the problem-space (JOOTSing), accessing “extended patterns of information” and higher computational power
- Irreversible transitions because reversals would reduce evolutionary advantage / computational power attained by the aggregates

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