Measuring Spatiotemporal Coordination in a Modular Robotic System

Mikhail Prokopenko[1], Vadim Gerasimov[1] and Ivan Tanev[2]

[1]CSIRO Information and Communication Technology Centre, Locked bag 17, North Ryde, NSW 1670, Australia
[2]Department of Information Systems Design, Doshisha University, 1-3 Miyakodani, Tatara, Kyotanabe, Kyoto 610-0321, Japan
ATR Network Informatics Laboratories, 2-2-2 Hikaridai, "Keihanna Science City", Kyoto 619-0288, Japan
mikhail.prokopenko@csiro.au

Abstract

In this paper we propose and verify spatiotemporal measures of coordination in a modular robotic system. These information-theoretic measures estimate the generalized "correlation entropy" K_2 and the generalized excess entropy E_2 computed over a multivariate time series of actuators' states, while standard deviation of E_2 is minimized over both space and time. Actuators are shown to be well-coordinated in individuals with fastest locomotion. These results support our conjecture that direct fitness functions can be approximated with generic selection pressures, leading towards *information-driven evolutionary design*.

Introduction

Modular Robotics is one of the most rapidly growing applications of Artificial Life, supported by advances in sensor and actuator technologies, multi-agent control theory, and studies of self-organisation. Robots built of several similar building blocks (modules) are attractive due to high versatility in their shapes, locomotion modes, tasks, and manipulation abilities (Bojinov et al., 2002; Yim et al., 2003; Tanev et al., 2005; Tanev, 2005; Dorigo, 2005). This multi-faceted versatility increases robustness, adaptability, and scalability required in many practical systems, ranging from search and rescue to space exploration. These requirements are often observed in biological systems. Unlike traditional engineering and robotic systems which are often brittle and inflexible, such systems are not assembled out of separately designed parts attached together at a later stage — they co-evolve (Goldsmith and Miller, 2003). The result is a symbiotic scalable system where components can be reused for other purposes and take on multiple roles, increasing robustness and adaptivity on multiple levels: from a cell to an ant colony to social systems.

One principled way to address these requirements is to distribute sensing, actuation and computational capabilities throughout the modular robotic system to form a complex multi-agent network, enabling the desired responses to self-organize within the complex system, without central control. The main challenge is a design methodology for systematically inter-connecting a set of global system-level tasks, functions, etc. with sensors, behaviours, and actuators of localized agents. Such a methodology should enable a co-evolution of multiple agents (robotic modules, network nodes, swarm elements, etc.), fitting the selection criteria. An example of a selection pressure is the acquisition of information from the environment: there is some evidence that pushing the information flow to the information-theoretic limit (i.e., maximization of information transfer) can give rise to intricate behaviour, induce a necessary structure in the system, and ultimately be responsible for adaptively reshaping the system (Klyubin et al., 2004; Klyubin et al., 2005). A comprehensive approach, nevertheless, should include multiple design/selection pressures, including: stability of multi-agent hierarchies (Prokopenko et al., 2005b); robustness or "noise rejection" (El-Samad et al., 2005); efficiency of perception-action loops (Klyubin et al., 2004); efficiency of computation (computational complexity); efficiency of communication topologies (Prokopenko et al., 2005a); efficiency of locomotion and distributed actuation (Der et al., 1999; Tanev et al., 2005; Tanev, 2005). Each of these pressures can be represented in two ways — either via task-specific objectives or via generic information-theoretic fitness functions. Direct evolutionary computation using task-specific objectives is now a typical approach, and the solutions obtained by this method reflect success of hand-crafting fitness functions by human designers. We propose here another methodology: *information-driven evolutionary design*, which uses fitness functions according to generic information-theoretic criteria, e.g., minimization of the variance of the rule-space's entropy (Wuensche, 1999; Prokopenko et al., 2005b), maximization of information transfer in perception-action loops (Klyubin et al., 2004; Klyubin et al., 2005), minimization of Boltzmann entropy in swarm-bots' states (Baldassarre et al., 2005), etc. The solutions obtained by information-driven evolution can be judged by their degree of approximation to the direct-evolution results. A good approximation will indicate that the chosen criteria capture the information-theoretic core of selection pressures, leading to "taskless adaptation". The identification of possible intrinsic fitness criteria is related

to the work of Der *et al.* on self-organization of agent behaviors from domain-invariant principles, e.g., homeokinesis (Der et al., 1999).

This paper models one specific step towards a comprehensive approach to information-driven evolutionary design: information-theoretic measures of coordination in a modular robotic system. In order to study such measures in a transparent way, without obscuring it with other aspects such as sensing, computation, etc., we select a sufficiently simple system: a modular limbless, wheelless snake-like robot (Snakebot) (Tanev et al., 2005; Tanev, 2005) without sensors. The only design goal of Snakebot's evolution, reported by Tanev and his colleagues (Tanev et al., 2005; Tanev, 2005), is fastest locomotion. Our immediate goal is information-theoretic approximation of this direct evolution. Specifically, we construct measures of spatiotemporal coordination of distributed actuators used by a Snakebot in locomotion. The measures are based on the generalized "correlation entropy" K_2 (a lower bound of Kolmogorov-Sinai entropy) and its excess entropy E_2 computed over a multivariate time series of actuators' states. Studying entropy dynamics of over space and time allows us to identify Snakebots with most coordinated actuators. Our experiments confirmed that maximal coordination (measured information-theoretically) is achieved synchronously with fastest locomotion (a direct measure).

The following Section briefly reviews some relevant background results in modular robotics. Section describes the Snakebot and the proposed measures, followed by experimental set-up and evolution-tracing results (Section), and conclusions (Section).

Background and Motivation

Several control algorithms for metamorphic robotic systems capable of spatial reconfiguration, exemplifying the use of emergent behaviour, are described by Bojinov *et al.* (Bojinov et al., 2002): the multi-agent control achieves suitable reconfiguration as a "side-effect" of creating a structure with the properties (structural, morphological, etc.) required for a global task. Their agents (modules) have limited computational capabilities, communicating only with immediate neighbors, and using local rules to produce adequate multi-agent control algorithms. Dorigo (Dorigo, 2005) describes an experiment in swarm robotics (SWARM-BOT) which also complements standard self-reconfigurability with task-dependent cooperation. Small autonomous mobile robots (s-bots) physically aggregate into specific shapes enabling the collective structure (a swarm-bot) to perform specific functions beyond capabilities of a single module. The swarm-bot forms as a result of self-organization "rather than via a global template and is expected to move as a whole and reconfigure along the way when needed" (Dorigo, 2005). One basic ability of a swarm-bot, immediately relevant to our research, is *coordinated motion* emerging when the con-

stituent independently-controlled modules coordinate their actions in choosing a common direction of motion. *Locomotion* is an essential skill in biological organisms, typically defined (e.g., Wikipedia) as "self-powered, patterned motion of limbs or other anatomical parts by which an individual customarily moves itself from place to place". Our focus, motivated by studies of coordination in a modular robotic system, is on how much locomotion can indeed be "patterned" in an aggregated structure.

Regardless of an environment (aquatic, terrestrial or aerial), locomotion is achieved by applying forces generated by the rhythmic contraction of muscles attached to limbs, wings, fins, etc. It was observed that typically, a locomotory gait is efficient when all the involved muscles contract and extend with the same frequency and different phases (Bourquin, 2005). For example, Yim *et al.* (Yim et al., 2003) investigated a snake-like (serpentine) sinusoid gait, where forward motion is essentially achieved by propagating a waveform travelling down the length of the chain. Tanev and his colleagues (Tanev et al., 2005; Tanev, 2005) demonstrated emergence of side-winding locomotion (defined as locomotion predominantly perpendicular to the long axis of a snake-like modular robot, described in the next section), providing superior speed characteristics for considered morphology as well as adaptability to challenging terrain environments and partial damage.

Ijspeert *et al.* (Ijspeert et al., 2005) systematically studied salamander locomotion for a number of gaits. The swimming is based on "axial undulations in which rostrocaudal waves with a piece-wise constant wavelength are propagated along the whole body with limbs folded backwards". On ground, the salamander uses the stepping gait, with the body making S-shaped standing waves with nodes at the girdles, while the limbs are coordinated with the bending of the body to increase the stride length. Sometimes, for instance when trying to rapidly escape in grass, the salamander uses crawling (a serpentine gait with limbs folded against the body) as a third locomotion mode. Ijspeert *et al.* considered a number of scenarios where oscillations in a multi-segment chain, starting from random initial states, rapidly evolve to travelling or/and standing waves. This work related salamander locomotion to coordinated patterns of rhythmic neural activity, concluding that "from a dynamical systems point of view, locomotion becomes the limit cycle behavior of the controller-body-environment system" (Ijspeert et al., 2005). This is particularly interesting in a multi-agent dynamical system such as a modular robot, where locomotion is a result of modules' self-organisation. Is it then possible to capture the relation between locomotion and rhythmic inter-modular coordination via spatiotemporal measures?

Before presenting our approach to this question, we note a related investigation of Baldassarre *et al.* (Baldassarre et al., 2005), who characterized coordinated motion in a swarm collective as a self-organized activity, and measured the in-

creasing organization of the group on the basis of Boltzmann entropy. This study had the goal of clearly identifying and quantitatively describing the specific self-organizing mechanisms operating within modular robotic systems such as swarms. In particular, the emergent *common direction* of motion, with the chassis orientations of the robots spatially aligned, allows the group to achieve high coordination. Baldassarre *et al.* proposed a method to capture this spatial alignment via Boltzmann entropy — by dividing the state space of the elements of the system (modules) into cells (e.g., cells of $45°$ each, corresponding to chassis orientations), measuring the number of elements in each cell for a given macrostate m, computing the number w_m of microstates that compose m, and calculating Boltzmann entropy of the macrostate as $E_m = k \, ln[w_m]$, where k is a scaling constant. This constant is set to the inverse of the maximum entropy which is equal to the entropy of the macrostate where the all elements are equally distributed over the cells. The results indicate that, "independently of the size of the group, the disorganization of the group initially decreases with an increasing rate, then tends to decrease with a decreasing rate, and finally reaches a null value when all the robots have the same orientation" (Baldassarre et al., 2005).

In this work, we attempt to advance from a purely spatial characterization (such as Boltzmann entropy of a macrostate distributing chassis orientations over the cells) to a spatiotemporal measure. The entropy measure proposed in our work is intended to capture not only spatial alignment of different modules, but account also for temporal dependencies among them, such as travelling or standing waves in multisegment chains observed by Ijspeert *et al.*. Since we intend to use this measure in the information-driven evolution as a (partial) fitness function, it should ideally inter-relate spatial and temporal components.

Spatiotemporal Coordination of Actuators

Snakebot is simulated as a set of identical spherical morphological segments ("vertebrae"), linked together via universal joints. All joints feature identical (finite) angle limits, and each joint has two attached actuators ("muscles"). In the initial, standstill, position of Snakebot the rotation axes of the actuators are oriented vertically (vertical actuator) and horizontally (horizontal actuator). These actuators perform rotation of the joint in the horizontal and vertical planes respectively. No anisotropic (directional) friction between the morphological segments and the surface is considered. Open Dynamics Engine (ODE) was chosen to provide a realistic simulation of the mechanics in applying forces to the segments of Snakebot. Given this representation, the task of designing the fastest Snakebot locomotion can be rephrased as developing temporal patterns of desired turning angles of horizontal and vertical actuators for each joint, maximizing the overall speed. Previous experiments of evolvable locomotion gaits with fitness measured as ei-
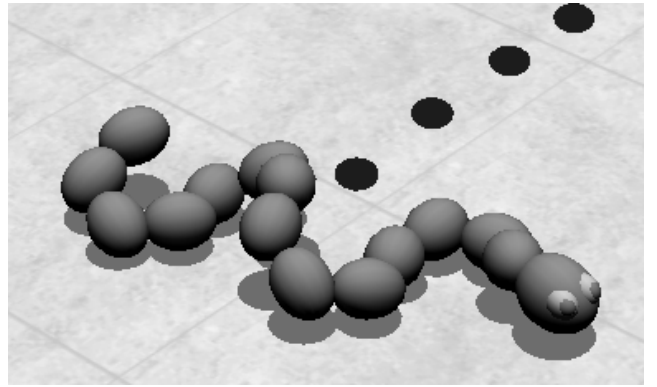

Figure 1: The dark trailing circles show the trajectory of the Snakebot's center of mass.

ther velocity in any direction or velocity in forward direction (Tanev et al., 2005) indicated that side-winding locomotion — locomotion predominantly perpendicular to the long axis of Snakebot (Figure 1) — provides superior speed characteristics for the considered morphology of Snakebot (inspired by the efficient side-winding locomotion of the rattlesnake Crotalus cerastes). The actuators states (horizontal and vertical turning angles) are constrained by the interactions between segments and the terrain. It is precisely the *actual turning angles* that provide an underlying time series for our information-theoretic analysis: horizontal turning angles $\{x_t^i\}$ and vertical turning angles $\{y_t^i\}$ at time t, where i is the actuator index, S is the number of joints, $1 \le i \le S$, and T is the considered time interval, $1 \le t \le T$. Since we deal with actual rather than ideal turning angles, the underlying dynamics in the phase-space may include both periodic and chaotic orbits.

We intend to estimate "irregularity" for each of the multivariate time series $\{x_t^i\}$ and $\{y_t^i\}$. Each of these time series, henceforth denoted for generality $\{v_t^i\}$, contains both spatial and temporal patterns, and minimizing the irregularity over both space and time dimensions should ideally uncover the extent of spatiotemporal coordination among actuator states.

For any given actuator i, a simple characterisation of the "regularity" of the time series $\{v_t\}$ is provided by the autocorrelation function. However, the auto-correlation is limited to measuring only linear dependencies. We consider instead a more general approach. One classical measure is the Kolmogorov-Sinai (KS) entropy, also known as metric entropy (Kolmogorov, 1959): it is a measure for the rate at which information about the state of the system is lost in the course of time. In other words, it is an entropy per unit time, an entropy rate or entropy density. Suppose that the $d-$dimensional phase space is partitioned into boxes of size r^d. Let $P_{i_0 \ldots i_{d-1}}$ be the joint probability that a trajectory is in box i_0 at time 0, in box i_1 at time Δt, ..., and in box i_{d-1} at time $(d-1)\Delta t$, where Δt is the time interval between measurements on the state of the system (in our case, we may

assume $\Delta t = 1$, and omit the limit $\Delta t \to 0$ in the following definitions). The KS entropy is defined by

$$K = -\lim_{r \to 0} \lim_{d \to \infty} \frac{1}{d\Delta t} \sum_{i_0 \ldots i_{d-1}} P_{i_0 \ldots i_{d-1}} \ln P_{i_0 \ldots i_{d-1}} \;, \quad (1)$$

and more precisely, as a supremum of K on all possible partitions. This definition has been generalized to the order-q Rényi entropies K_q (Rényi, 1970):

$$K_q = -\lim_{\Delta t \to 0} \lim_{r \to 0} \lim_{d \to \infty} \frac{1}{d\Delta t(q-1)} \ln \sum_{i_0 \ldots i_{d-1}} P^q_{i_0 \ldots i_{d-1}} \;. \quad (2)$$

It is well-known that $K = 0$ in an ordered system, K is infinite in a random system, and K is a positive constant in a deterministic chaotic system. Grassberger and Procaccia (Grassberger and Procaccia, 1983) considered the "correlation entropy" K_2 in particular, and capitalized on the fact $K \geq K_2$ in establishing a sufficient condition for chaos $K_2 > 0$. For a univariate time series $\{v_t\}$ (the actuator i is given), Grassberger and Procaccia algorithm estimates the entropy rate K_2 as follows:

$$K_2 = \lim_{r \to 0} \lim_{d \to \infty} \lim_{T \to \infty} \ln \frac{C_d(T,r)}{C_{d+1}(T,r)} \;, \quad (3)$$

where $C_d(r)$ is the correlation integral:

$$C_d(T,r) = \frac{1}{(T-1)T} \sum_{l=1}^{T} \sum_{j=1}^{T} \Theta(r - \|\vec{V}_l - \vec{V}_j\|) \;. \quad (4)$$

Here Θ is the Heaviside function (equal to 0 for negative argument and 1 otherwise), and the vectors $\vec{V}_l$ and $\vec{V}_j$ contain elements of the observed time series $\{v(t)\}$, "converting" or "reconstructing" the dynamical information in one-dimensional data to information in the d-dimensional embedding space: $\vec{V}_k = (v_k, v_{k+1}, v_{k+2}, \ldots, v_{k+d-1})$ (Takens, 1981). The norm $\|\vec{V}_l - \vec{V}_j\|$ is the distance between the vectors in the d-dimensional space, e.g., the maximum norm:

$$\|\vec{V}_l - \vec{V}_j\| = \max_{\tau=0}^{d-1}(v_{l+\tau} - v_{j+\tau}) \quad (5)$$

Put simply, $C_d(r)$ computes the fraction of pairs of vectors in the d-dimensional embedding space that are separated by a distance less than or equal to r. In order to eliminate auto-correlation effects, the vectors in Equation (4) should be chosen to satisfy $|l - j| > L$, for some positive L, and at the very least $l \neq j$.

For our analysis we need to introduce a spatial dimension (across multiple Snakebot's actuators). Carretero-González et al. argued that a direct application of univariate (temporal) delay reconstructions would fail for spatiotemporal systems (Carretero-González et al., 2000), and suggested that a spatiotemporal reconstruction can

be obtained by replacing the time-delay vectors $\vec{V}_k = (v_k, v_{k+1}, v_{k+2}, \ldots, v_{k+d-1})$ by the spatiotemporal delay vectors $\vec{V}^i_k = (\vec{v}^i_k, \vec{v}^{i+1}_k, \vec{v}^{i+2}_k, \ldots, \vec{v}^{i+\vec{d}_s-1}_k)$ whose elements are time-delay vectors $\vec{v}^i_k = (v^i_k, v^i_{k+1}, v^i_{k+2}, \ldots, v^i_{k+d_t-1})$, and the spatial index i is fixed. The overall embedding dimension for such a spatiotemporal reconstruction is $d = d_s d_t$, where d_s and d_t denote the spatial and temporal embedding dimensions, respectively (the standard temporal delay reconstruction is recovered by setting $d_s = 1$) (Carretero-González et al., 2000). The correlation integral $C_{d_s d_t}(T,S,r)$ can then be generalized to:

$$\frac{1}{(T-1)T(S-1)S} \sum_{l=1}^{T} \sum_{j=1}^{T} \sum_{g=1}^{S} \sum_{h=1}^{S} \Theta(r - \|\vec{V}^g_l - \vec{V}^h_j\|) \;. \quad (6)$$

The condition $|l - j| > L$ for time-delays is similarly extended to Snakebot actuator-proximity $|g - h| > M$, for an integer M, in order to exclude auto-correlation effects among closely coupled segments. It is less trivial, however, to generalize the entropy rate estimate (3). The spatiotemporal entropy density for cellular automata, for example, can be defined as $h^{d_s d_t} = \lim_{d_s \to \infty} h(d_s)/d_s$, where $h(d_s)$ is the entropy density of "patterns" ("blocks" or "words") $V(d_s, d_t)$ of spatial size d_s and time length d_t, given by

$$h(d_s) = \lim_{d_t \to \infty} \frac{-1}{d_t} \sum_{V(d_s, d_t)} p(V(d_s, d_t)) \ln p(V(d_s, d_t)) \;. \quad (7)$$

As noted by Boffetta *et al.* (Boffetta et al., 2002), the entropy $h^{d_s d_t}$ cannot be practically computed, and the spatiotemporal behavior can be obtained by studying either temporal sequences of increasing d_t for a fixed spatial extent D_s:

$$h^{d_t}(D_s) = \lim_{d_t \to \infty} \frac{-1}{d_t} \sum_{V(D_s, d_t)} p(V(D_s, d_t)) \ln p(V(D_s, d_t)) \quad (8)$$

or spatial sequences of increasing d_s for a given time D_t:

$$h^{d_s}(D_t) = \lim_{d_s \to \infty} \frac{-1}{d_s} \sum_{V(d_s, D_t)} p(V(d_s, D_t)) \ln p(V(d_s, D_t)) \quad (9)$$

Following this suggestion, we propose to estimate correlation entropy K_2 obtained from the multivariate time series with S actuators (joints) and T time steps, as

$$K_2^{d_t}(D_s, T, S, r) = \ln \frac{C_{d_s d_t}(T,S,r)}{C_{d_s(d_t+1)}(T,S,r)} \;, \quad (10)$$

for a fixed spatial extent D_s, and as

$$K_2^{d_s}(D_t, T, S, r) = \ln \frac{C_{d_s d_t}(T,S,r)}{C_{(d_s+1)d_t}(T,S,r)} \;. \quad (11)$$

for a given time D_t, where correlation integrals $C_{d_s d_t}(T,S,r)$ are defined by expression (6).

The correlation entropy K_2 (the generalized entropy rate) measures the irregularity or unpredictability of the system. A complementary quantity is the *excess entropy E* (Feldman and Crutchfield, 2003) — it may be viewed as a measure of the apparent memory or structure in the system. The generalized excess entropy E_2 is defined by considering how the finite-template (finite-delay or finite-extent) entropy rate estimates $K_2^{d_t}(D_s, T, S, r)$ and $K_2^{d_s}(D_t, T, S, r)$ converge to their asymptotic values $h^{d_t}(D_s)$ and $h^{d_s}(D_t)$. It is estimated as:

$$E_2^{d_t}(D_s, T, S, r) = \sum_{d_t=1}^{D_t} (K_2^{d_t}(D_s, T, S, r) - h^{d_t}(D_s)) , \quad (12)$$

for a fixed spatial extent D_s, and

$$E_2^{d_s}(D_t, T, S, r) = \sum_{d_s=1}^{D_s} (K_2^{d_s}(D_t, T, S, r) - h^{d_s}(D_t)) , \quad (13)$$

for a given time D_t. For regular locomotion the asymptotic values should be zero (while non-zero entropies would indicate non-periodicity, i.e. deterministic chaos).

Following this, we determine standard deviation σ^{d_t} of $E_2^{d_t}(D_s, T, S, r)$ over spatial extent D_s, and σ^{d_s} of $E_2^{d_s}(D_t, T, S, r)$ over time D_t. In well-coordinated Snakebots, different spatial extents D_s should "agree" on the temporal excess entropy. Similarly, different time delays D_t should not "disagree" in terms of the spatial excess entropy.

In summary, our estimation procedure involves three steps: 1) estimation of the correlation entropies K_2 (the generalized entropy rate) for a range of finite spatial extents and temporal delays; 2) estimation of the spatial and temporal excess entropies E_2 by integrating over the finite-template entropy rates; 3) calculation of standard deviations σ^{d_t} and σ^{d_s} of the temporal and spatial excess entropies E_2 over spatial and temporal delays respectively. The resultant measures target the divergence of global structural patterns.

Results

In this section we present experimental results tracing estimates of entropy rates $K_2^{d_t}(D_s, T, S, r)$ and $K_2^{d_s}(D_t, T, S, r)$, as well as their excess entropies, during evolution of a Snakebot. The Genetic Programming techniques employed in the evolution are described elsewhere (Tanev et al., 2005; Tanev, 2005). Snakebots evolve within a population of 200 individuals, and we trace the entropy rate of the best performer over a number of generation.

Figures 2 and 3 contrast (for vertical actuators) temporal entropy rates $K_2^{d_t}(D_s, T, S, r)$ of the first offspring and the final generation. Each figure shows temporal behaviour for multiple ranges (extents) of actuators, and their non-zero values indicate the amount of non-periodicity, achieved only asymptotically. For each fixed D_s, the excess entropy $E_2^{d_t}(D_s, T, S, r)$ integrates these "over-estimates" $K_2^{d_t}(D_s, T, S, r)$ over finite time delays d_t, and measures temporal structure in the system. The evolved solution has a

much better convergence as an ensemble of actuators across multiple spatial extents, producing smaller σ^{d_t}, highlighted by Figure 4. In other words, there is a convergence of the temporal excess entropies across the spatial dimension.

Figures 5 (the first offspring) and 6 (the final generation) compare spatial entropy rates $K_2^{d_s}(D_t, T, S, r)$ for multiple time-delays. Again, for each fixed D_t, the excess entropy $E_2^{d_s}(D_t, T, S, r)$ integrates the non-zero estimates $K_2^{d_s}(D_t, T, S, r)$ over finite spatial extents d_s, and measures the spatial structure in the system. The evolved solution converges better across multiple time-delays, generating smaller σ^{d_s} (Figure 7). This indicates that the spatial excess entropies become similar across time-delays as the Snakebot evolves. Since the direct-evolution targeted the overall velocity, the temporal excess entropies converge better than the spatial excess entropies — hence a slight increase in divergence of σ^{d_s} for later generations. The reason is that the Snakebot has 14 joints and is most coordinated at the mid-range $d_s = 7$. Extending beyond the middle leads to slightly more diverging entropy rates and excess entropies. Results for the horizontal actuators (which contribute less to overall locomotion than the vertical actuators) are very similar, and are omitted due to a lack of space.

Conclusions

We modelled a specific step towards a theory of information-driven evolutionary design: information-theoretic measures of coordination in a modular robotic system (Snakebot). The measures estimate the generalized "correlation entropies" K_2 computed over a time series of actuators' states, their excess entropies E_2, and standard deviations of E_2. Minimization of these deviations is linked with coordination over space and time. As expected, maximal coordination of actuators is achieved by individuals with fastest locomotion (a direct measure). Moreover, a fitness function rewarding coordination may sometimes be more suitable than a direct velocity-based measure. For example, a Snakebot trapped by obstacles may need to employ a locomotion gait with highly coordinated actuators but near-zero absolute velocity. An information-driven evolution minimizing both standard deviations σ^{d_t} and σ^{d_s} is likely to produce fast Snakebots with a higher degree of spatial regularity and modularity than those observed so far. Balancing between σ^{d_t} and σ^{d_s} allows the designers to choose between temporal and spatial regularity. A single spatiotemporal 2D excess entropy, described in (Feldman and Crutchfield, 2003), is interesting as well — it is likely to produce a smoother objective (fitness) function, at the expense of the balancing option. We agree with Baldassarre *et al.* (Baldassarre et al., 2005) that this type of analysis relating distributed coordination and self-organizing principles is rarely done in research on collective and modular robotics. The establishment of an adequate set of the information-theoretic criteria will eventually support a set of design guidelines for Artificial-Life systems.
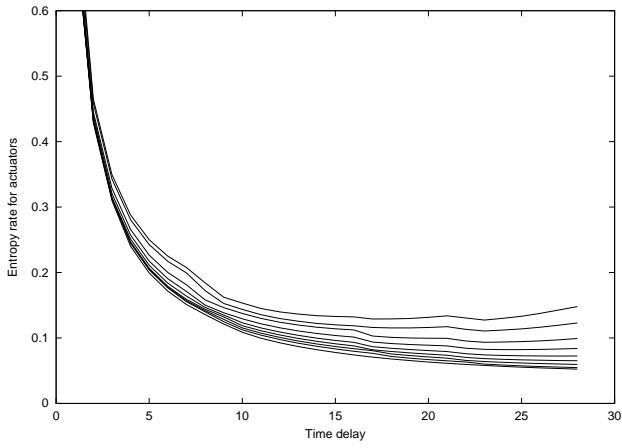
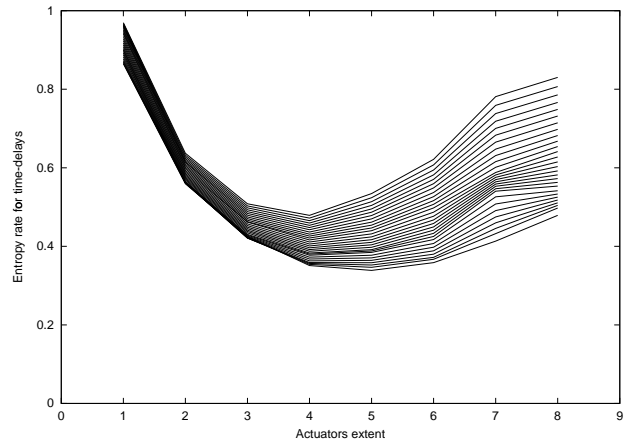Figure 2: The best first offspring. Vertical actuators: temporal entropy rate $K_2^{d_t}(D_s, T, S, r)$.

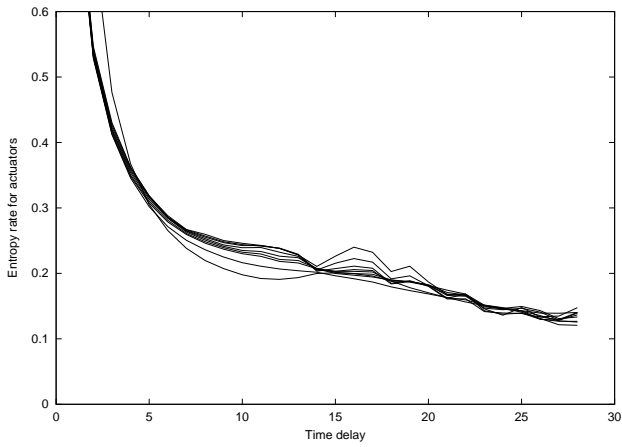Figure 5: The best first offspring. Vertical actuators: spatial entropy rate $K_2^{d_s}(D_t, T, S, r)$.

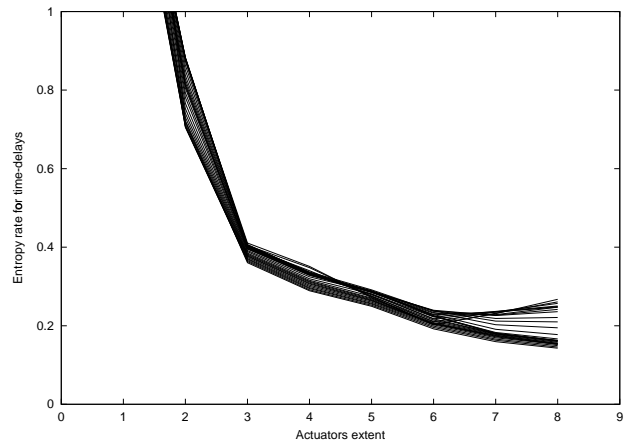Figure 3: The best evolved solution. Vertical actuators: temporal entropy rate $K_2^{d_t}(D_s, T, S, r)$.

Figure 6: The best evolved solution. Vertical actuators: spatial entropy rate $K_2^{d_s}(D_t, T, S, r)$.
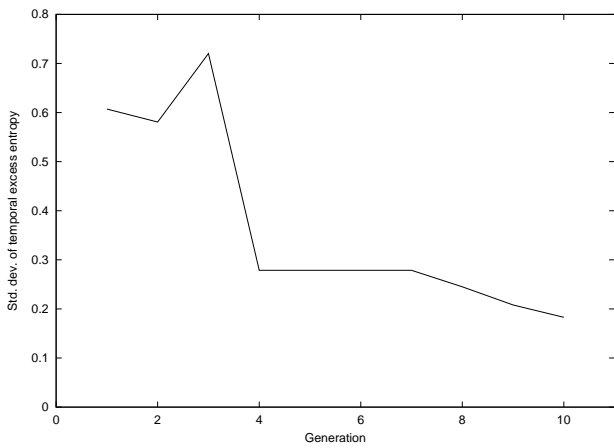

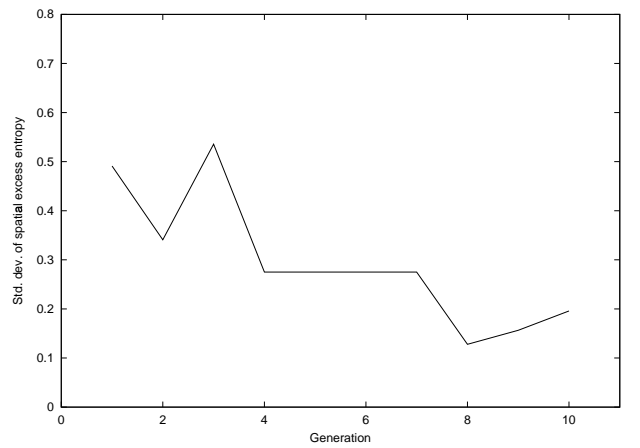Figure 4: Standard deviation σ^{d_t} of temporal excess entropy for vertical actuators.

Figure 7: Standard deviation σ^{d_s} of spatial excess entropy for vertical actuators.

References

Baldassarre, G., Parisi, D., and Nolfi, S. (2005). Measuring coordination as entropy decrease in groups of linked simulated robots. Preprint.

Boffetta, G., Cencini, M., Falcioni, M., and Vulpiani, A. (2002). Predictability: a way to characterize complexity. *Physics Reports*, 356:367–474.

Bojinov, H., Casal, A., and Hogg, T. (2002). Multiagent control of self-reconfigurable robots. *Artificial Intelligence*, 142:99–120.

Bourquin, Y. (2005). Self-organization of locomotion in modular robots. Master's thesis, Department of Informatics, University of Sussex, Brighton, U.K.

Carretero-González, R., Ørstavik, S., and Stark, J. (2000). Quasidiagonal approach to the estimation of lyapunov spectra for spatiotemporal systems from multivariate time series. *Phys Rev E Stat Phys Plasmas Fluids Relat Interdiscip Topics*, 62(5) Pt A:6429–6439.

Der, R., Steinmetz, U., and Pasemann, F. (1999). Homeokinesis - a new principle to back up evolution with learning. *Concurrent Systems Engineering Series*, 55: Computational Intelligence for Modelling, Control, and Automation:43–47.

Dorigo, M. (2005). Swarm-bot: An experiment in swarm robotics. In Arabshahi, P. and Martinoli, A., editors, *Proceedings of SIS 2005 – 2005 IEEE Swarm Intelligence Symposium*, pages 192–200. IEEE Press.

El-Samad, H., Kurata, H., Doyle, J., Gross, C., and Khammash, M. (2005). Surviving heat shock: Control strategies for robustness and performance. *Proceedings of the National Academy of Sciences*, 102(8):2736–2741.

Feldman, D. and Crutchfield, J. (2003). Structural information in two-dimensional patterns: Entropy convergence and excess entropy. *Physical Review E*, 67, 051104.

Goldsmith, R. and Miller, J. (2003). Cooperative co-evolution of robot control, sensor relevance and placement. In *Proccedings of EPSRC Evolvability and Sensor Evolution Symposium*. Birmingham, U.K.

Grassberger, P. and Procaccia, I. (1983). Estimation of the kolmogorov entropy from a chaotic signal. *Phys. Review A*, 28(4):2591.

Ijspeert, A., Crespi, A., and Cabelguen, J. (2005). Simulation and robotics studies of salamander locomotion. Applying neurobiological principles to the control of locomotion in robots. *Neuroinformatics*, 3(3):171–196.

Klyubin, A. S., Polani, D., and Nehaniv, C. L. (2004). Organization of the information flow in the perception-action loop of evolved agents. In *Proceedings of 2004 NASA/DoD Conference on Evolvable Hardware*, pages 177–180. IEEE Computer Society.

Klyubin, A. S., Polani, D., and Nehaniv, C. L. (2005). All else being equal be empowered. In Capcarrère, M., Freitas, A., Bentley, P., Johnson, C., and Timmis, J., editors, *Advances in Artificial Life, 8th European Conference, ECAL 2005, Canterbury, UK, September 5-9, 2005, Proceedings*, volume 3630 of *Lecture Notes in Computer Science*, pages 744–753. Springer.

Kolmogorov, A. (1959). Entropy per unit time as a metric invariant of automorphisms. *Doklady Akademii Nauk SSSR*, 124:754–755.

Prokopenko, M., Wang, P., Foreman, M., Valencia, P., Price, D., and Poulton, G. (2005a). On connectivity of reconfigurable impact networks in ageless aerospace vehicles. *Robotics and Autonomous Systems*, 53:36–58.

Prokopenko, M., Wang, P., Valencia, P., Price, D., Foreman, M., and Farmer, A. (2005b). Self-organizing hierarchies in sensor and communication networks. *Artificial Life*, 11:407–426.

Rényi, A. (1970). *Probability theory*. North-Holland.

Takens, F. (1981). Detecting strange attractors in turbulence. *Dynamical systems and turbulence*, 898:366.

Tanev, I. (2005). Learned mutation strategies in genetic programming for evolution and adaptation of simulated snakebot. *Genetic Evolutionary Computation - Gecco 2005, Proc.*, pages 687–694.

Tanev, I., Ray, T., and Buller, A. (2005). Automated evolutionary design, robustness, and adaptation of sidewinding locomotion of a simulated snake-like robot. *IEEE Transactions On Robotics*, 21:632–645.

Wuensche, A. (1999). Classifying cellular automata automatically: Finding gliders, filtering, and relating space-time patterns, attractor basins, and the z parameter. *Complexity*, 4(3):47–66.

Yim, M., Roufas, K., Duff, D., Zhang, Y., Eldershaw, C., and Homans, S. (2003). Modular reconfigurable robots in space applications. *Autonomous Robots*, 14:225–237.