Principle of the “super-efficiency”: Thermodynamic efficiency of self-organisation

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Computation and Thermodynamics
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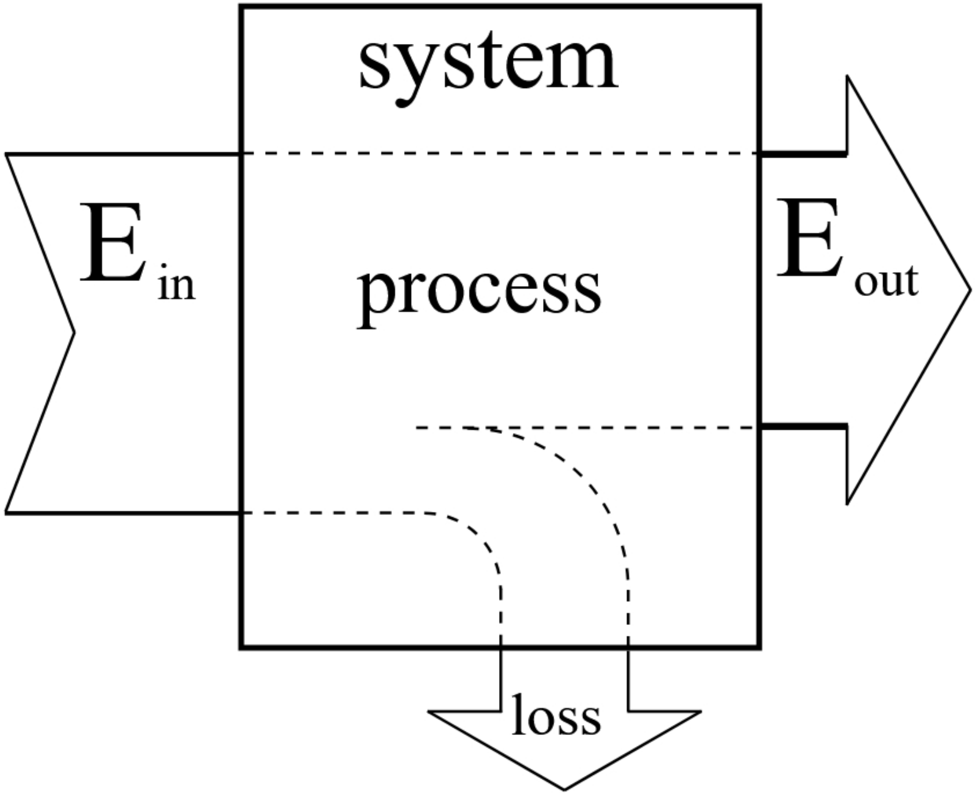
Outline

- Thermal vs thermodynamic efficiency
- Criticality and phase transitions
- Fisher information: information theory and thermodynamics
- Case studies:
  - collective / swarming motion (*Physical Review E*, 2018)
  - urban dynamics (*Royal Society Open Science*, 2018)
  - epidemic dynamics (*Royal Society Interface Focus*, 2018)
  - Curie-Weiss Ising model (*Entropy*, 2021)

Thermal efficiency



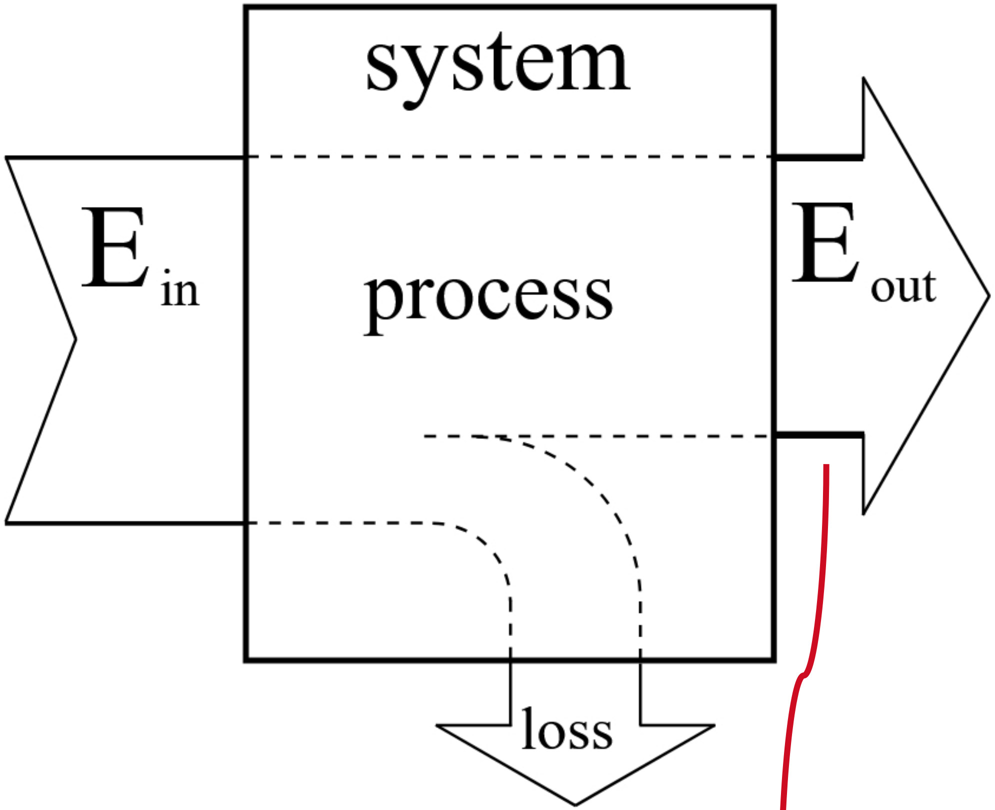
$$Q_{in} = |W_{\text{out}}| + |Q_{\text{out}}|$$

$$\eta_{\text{th}} \equiv \frac{\text{benefit}}{\text{cost}}$$

$$\eta_{\text{th}} \equiv \frac{|W_{\text{out}}|}{Q_{\text{in}}}$$

https://en.wikipedia.org/wiki/Thermal_efficiency

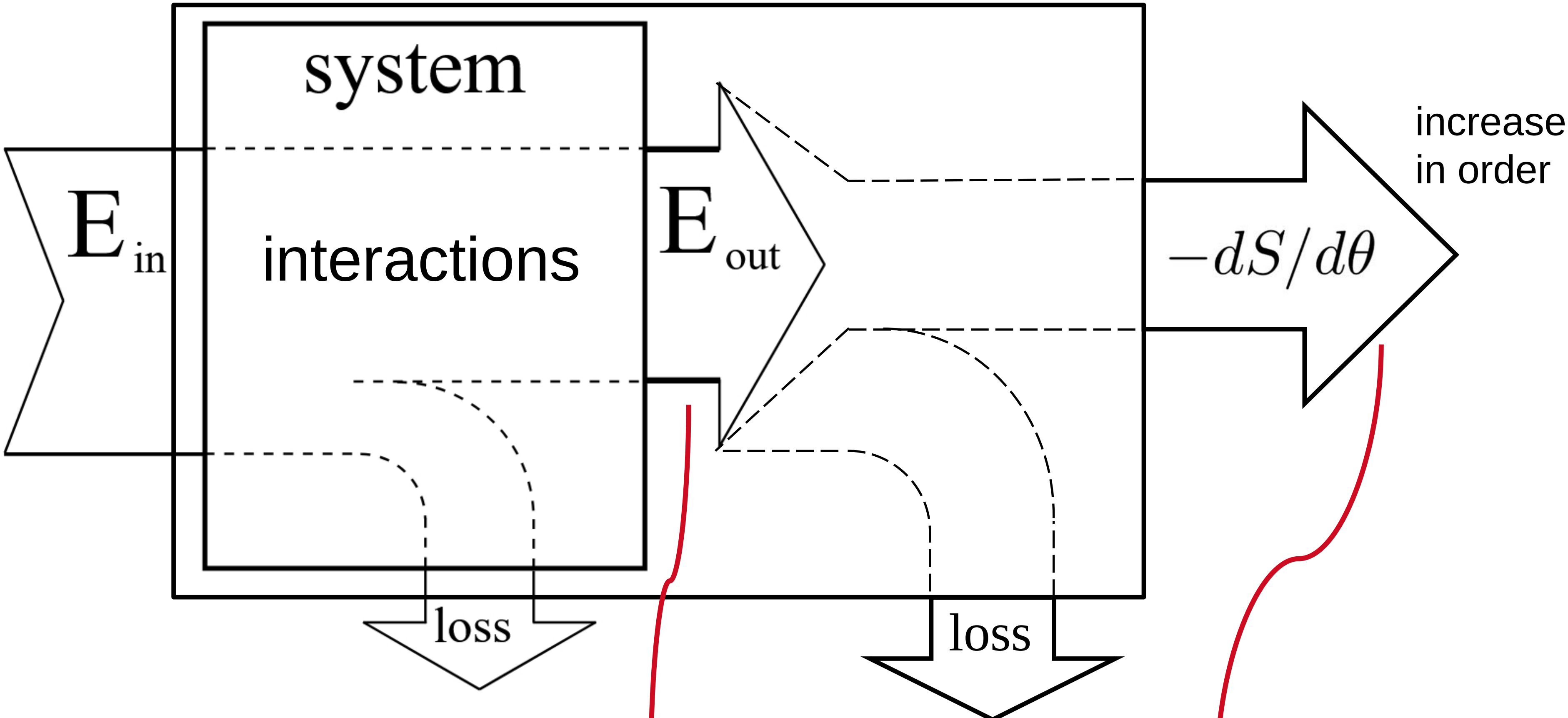
Thermal efficiency



$$\eta_{\mathrm{th}} \equiv \frac{|W_{\mathrm{out}}|}{Q_{\mathrm{in}}}$$

Thermodynamic efficiency

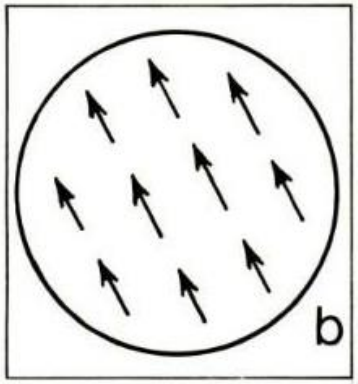
self-organisation



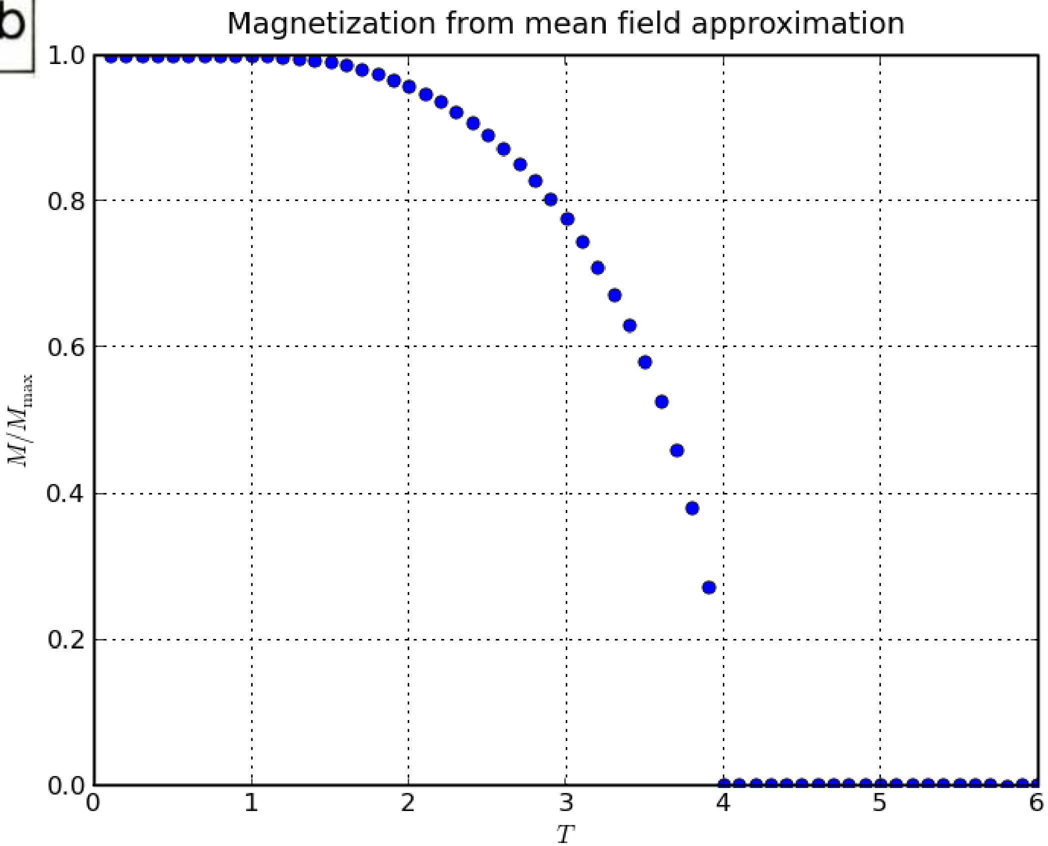
$$\eta_{\mathrm{th}} \equiv \frac{|W_{\mathrm{out}}|}{Q_{\mathrm{in}}}$$

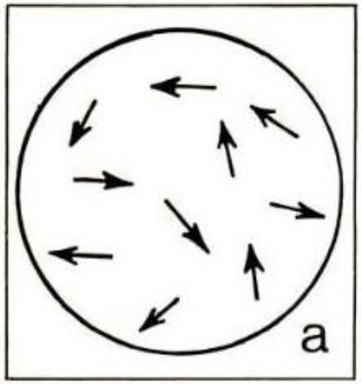
$$\eta \equiv \frac{-dS/d\theta}{d\langle \beta W_{gen}\rangle / d\theta}$$

Phase transitions and order parameters



anisotropic
“coherent”

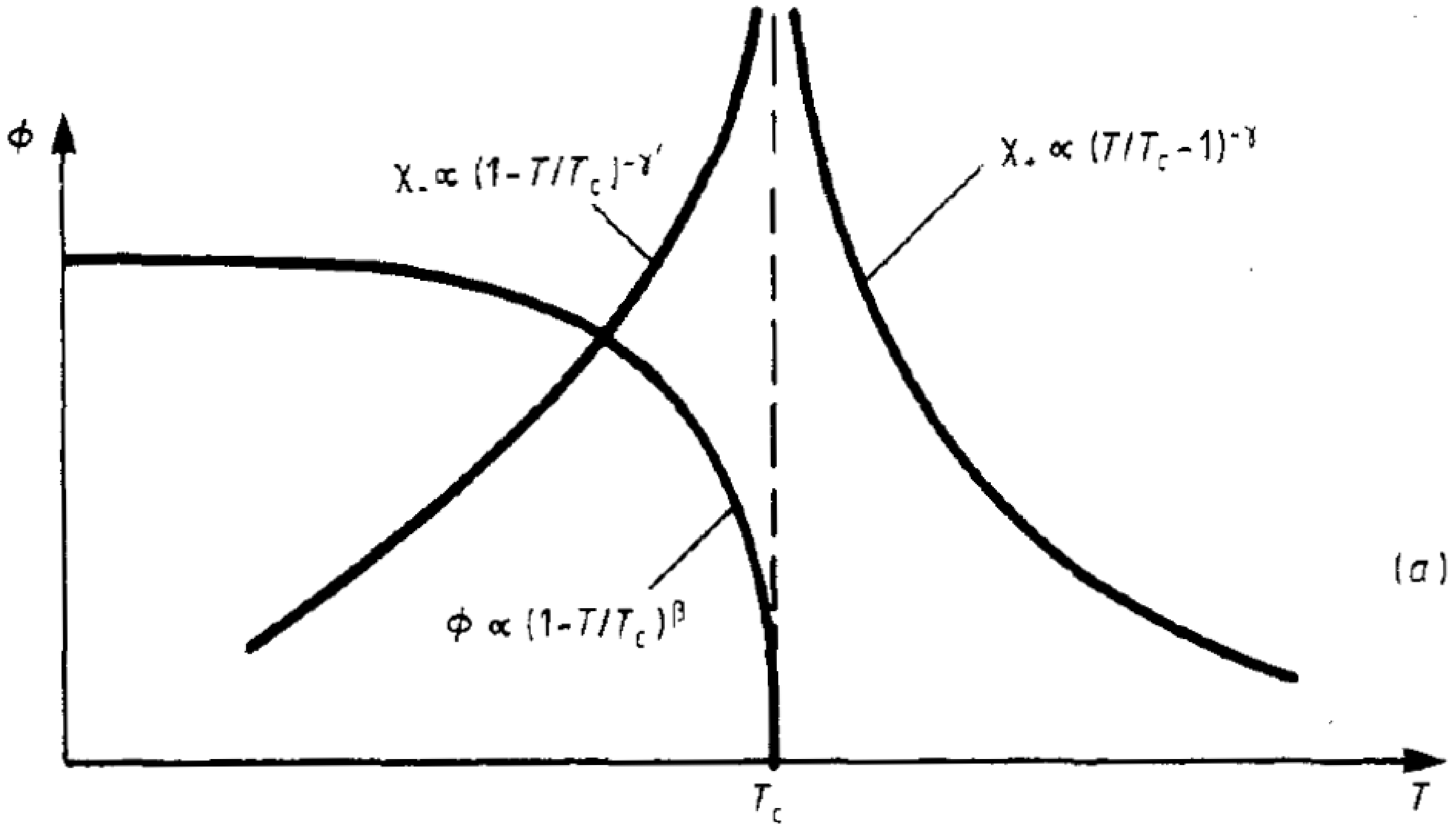




isotropic
“disordered”

Derivative of order parameter (divergence)

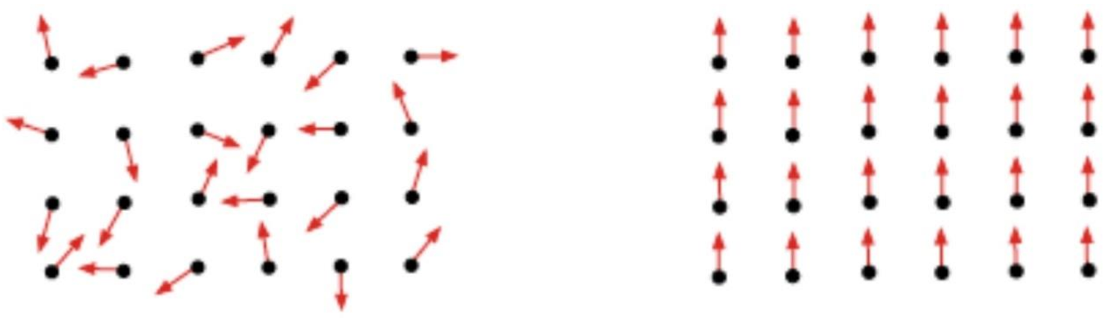
K Binder **(1987)**



Fisher Information and sensitivity

A way of measuring the amount of information that an observable random variable X has about an unknown parameter θ

$$F(\theta) = \int_x \left(\frac{\partial \ln(p(x|\theta))}{\partial \theta} \right)^2 p(x|\theta) dx$$



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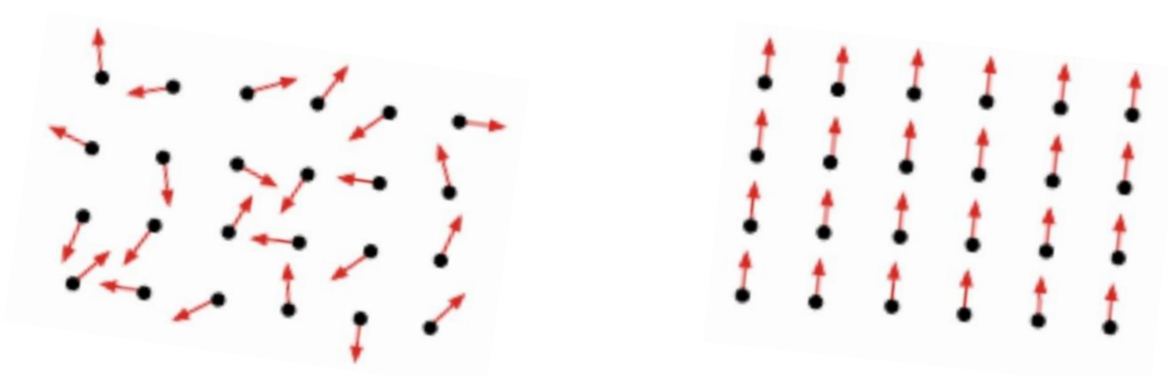

Fisher Information and order parameters

$$G(T,\theta_i) = U(S,\phi_i) - TS - \phi_i\theta_i$$

$$\boxed{F_{ij}(\theta) = \beta\frac{\partial\phi_i}{\partial\theta_j}}$$

Fisher information matrix

Rate of change of the order parameter

M. Prokopenko, J. T. Lizier, O. Obst, X. R. Wang, Relating Fisher information to order parameters, *Physical Review E*, 84, 041116, 2011.

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Fisher Information and generalised work

- A way of measuring the amount of information that an observable random variable X has about an unknown parameter θ

$$F(\theta) = \int_x \left(\frac{\partial \ln(p(x|\theta))}{\partial \theta} \right)^2 p(x|\theta) dx$$

- Fisher information is proportional to the curvature of the work in quasi-static processes

$$F(\theta) = -\frac{d^2 \langle \beta W_{gen} \rangle}{d\theta^2}$$

E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, *Physical Review E,* 97, 012120, 2018.

Thermodynamic efficiency

The reduction in uncertainty (the increase in order) from an expenditure of work for a given value of control parameter

$$\eta \equiv \frac{-dS/d\theta}{d\langle \beta W_{gen} \rangle / d\theta}$$

in thermodynamic terms

E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, *Physical Review E,* 97, 012120, 2018.

Thermodynamic efficiency

$$F(\theta) = -\frac{d^2\langle \beta W_{gen}\rangle}{d\theta^2}$$

$$\eta \equiv \frac{-dS/d\theta}{d\langle \beta W_{gen}\rangle / d\theta} = \frac{-dS/d\theta}{\int_{\theta}^{\theta^*} F(\theta')d\theta'}$$

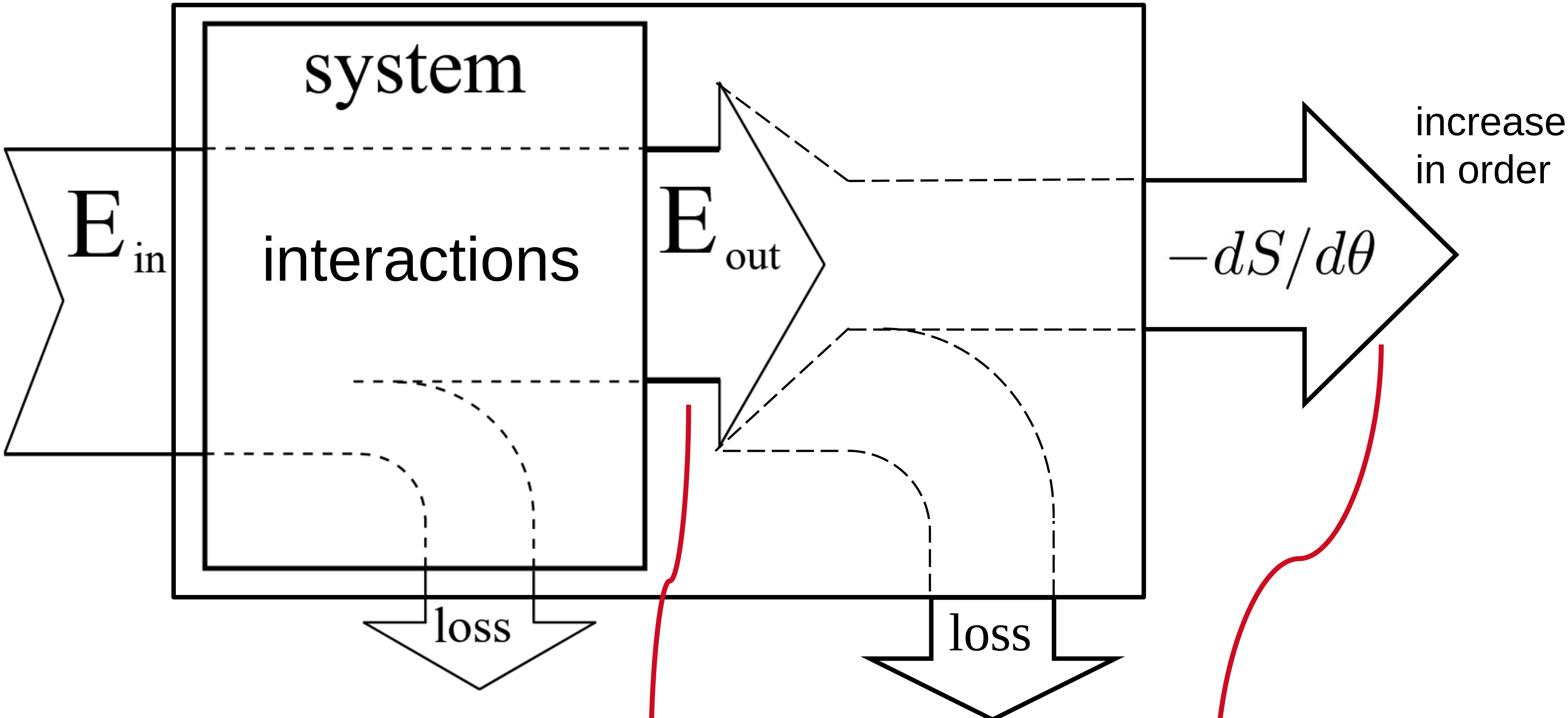
in thermodynamic terms

in computational terms

E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, *Physical Review E,* 97, 012120, 2018.

Thermodynamic efficiency

self-organisation

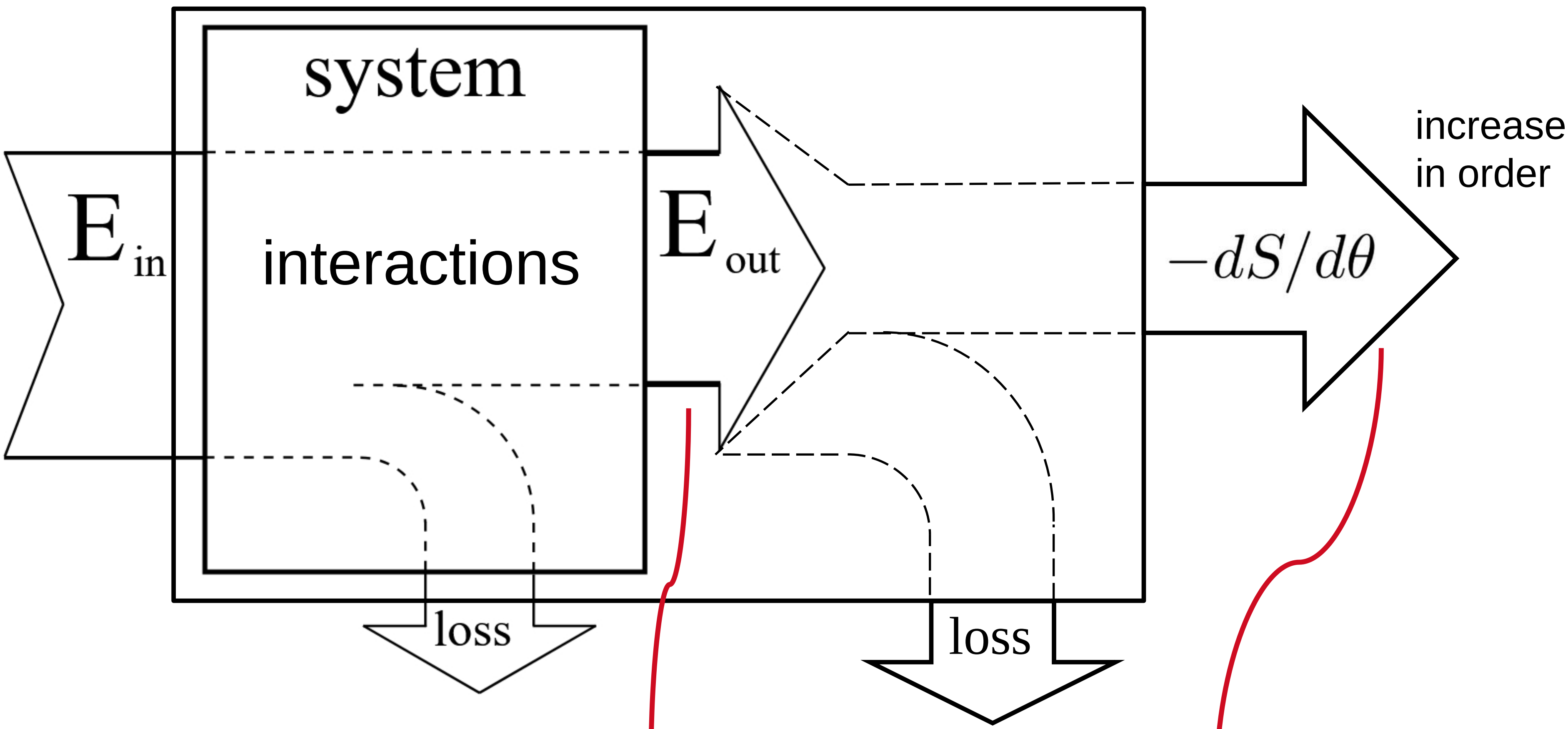


$$\eta_{\mathrm{th}} \equiv \frac{|W_{\mathrm{out}}|}{Q_{\mathrm{in}}}$$

$$\eta \equiv \frac{-dS/d\theta}{d\langle \beta W_{gen}\rangle/d\theta}$$

Thermodynamic efficiency

self-organisation



$$\eta_{\rm th} \equiv \frac{|W_{\rm out}|}{Q_{\rm in}}$$

$$\eta \equiv \frac{-dS/d\theta}{\int_{\theta}^{\theta^*} F(\theta')d\theta'}$$

1. A dynamical model of collective motion

VOLUME 92, NUMBER 2 PHYSICAL REVIEW LETTERS week ending 16 JANUARY 2004

Onset of Collective and Cohesive Motion

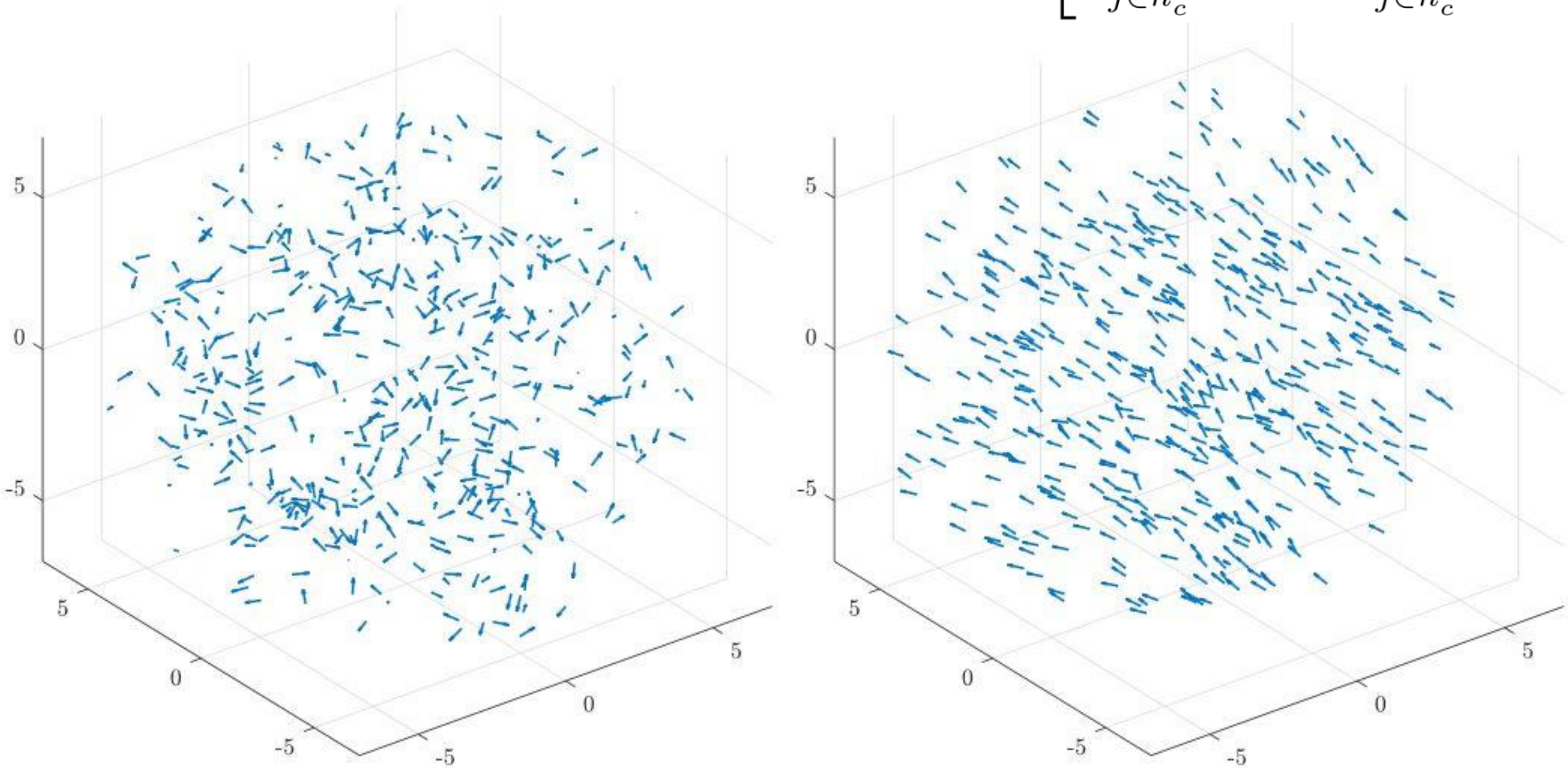
Guillaume Grégoire and Hugues Chaté
CEA–Service de Physique de l'État Condensé, CEN Saclay, 91191 Gif-sur-Yvette, France
Pôle Matière et Systèmes Complexes, CNRS FRE 2348, Université de Paris VII, Paris, France
(Received 12 August 2003; published 15 January 2004)

$$\mathbf{x_i}(t+1) = \mathbf{x_i}(t) + \mathbf{v_i}(t)$$

$$\mathbf{v_i}(t+1) = v_0 \Theta \left[\underbrace{a}_{\text{alignment}} \sum_{j \in n_c^i} \mathbf{v_j}(t) + \underbrace{b}_{\text{cohesion}} \sum_{j \in n_c^i} f_{ij} + \underbrace{n_c \boldsymbol{\eta}_\mathbf{i}}_{\text{perturbation}} \right]$$

Swarming (collective) motion

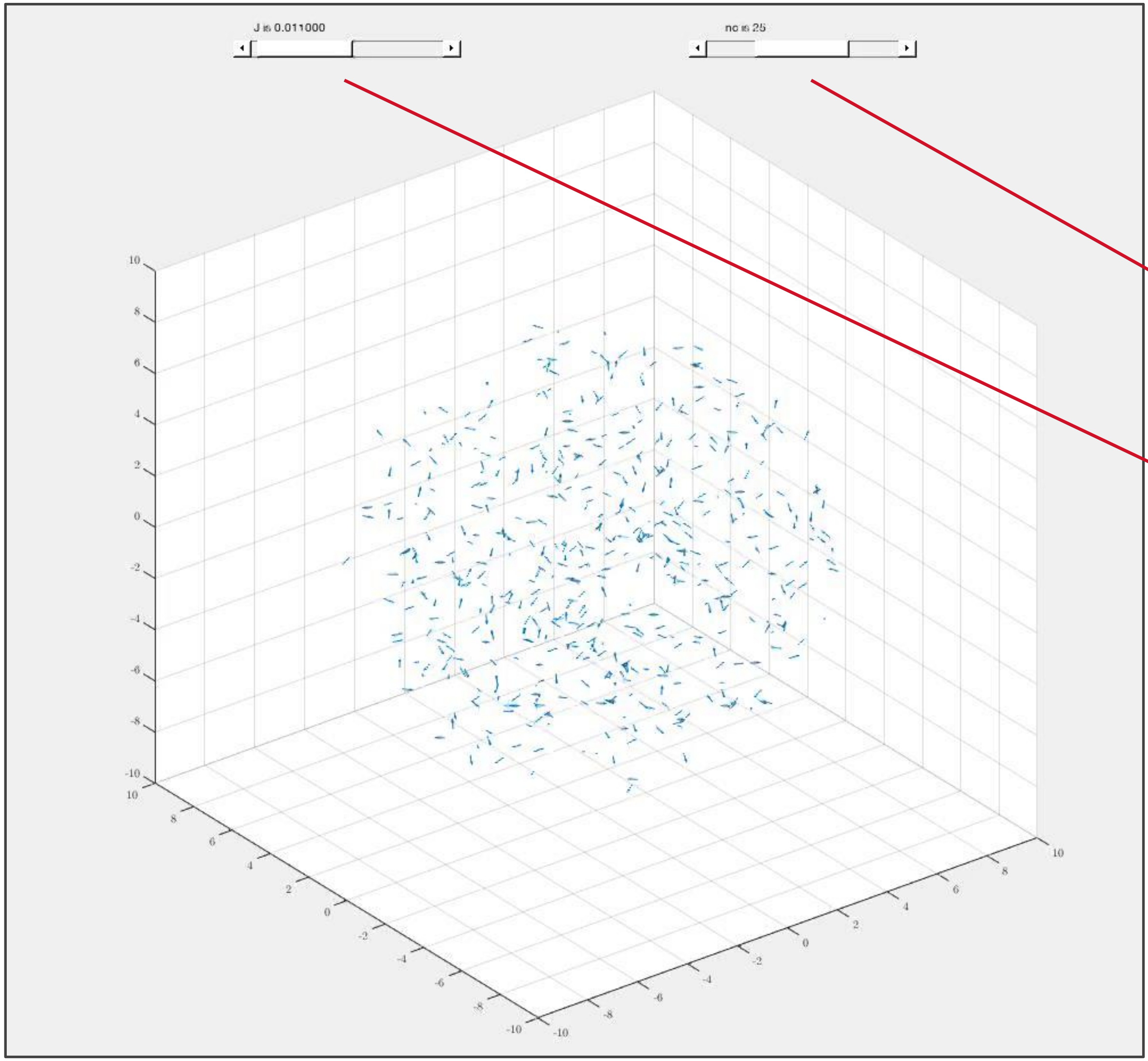
$$\mathbf{v_i}(t+1) = v_0 \Theta \left[a \sum_{j \in n_c^i} \mathbf{v_j}(t) + b \sum_{j \in n_c^i} f_{ij} + n_c \boldsymbol{\eta}_\mathbf{i} \right]$$



(a) $J = 0.001,\ n_c = 20$ (b) $J = 0.2,\ n_c = 20$

E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, *Physical Review E,* 97, 012120, 2018.

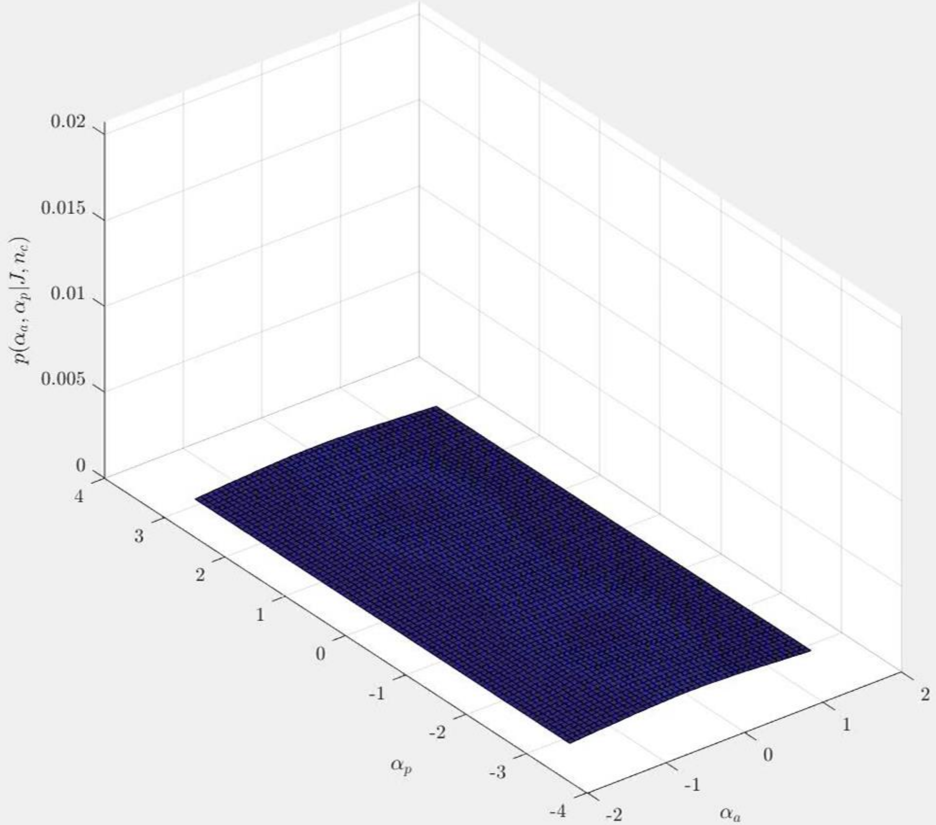
Varying control parameters

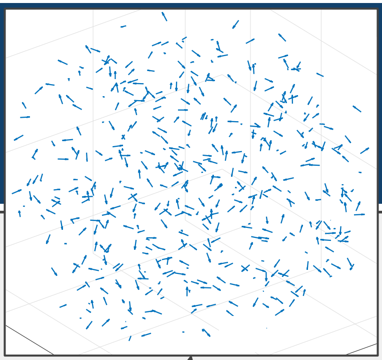


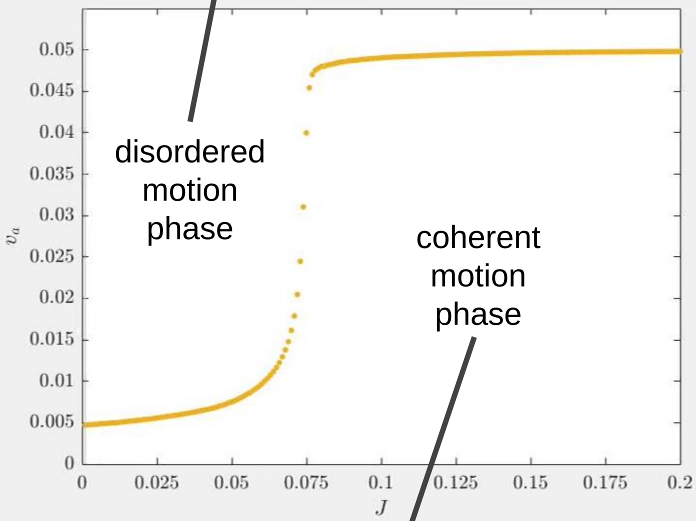
a kinetic phase transition driven by

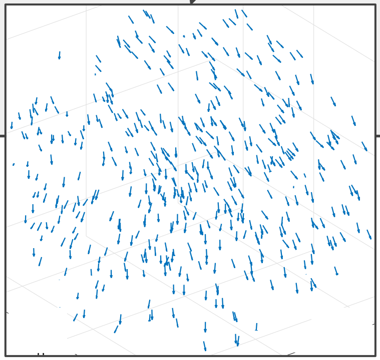
- nearest neighbours N_c
- alignment strength $J = v_0 a$

The two kinetic phases



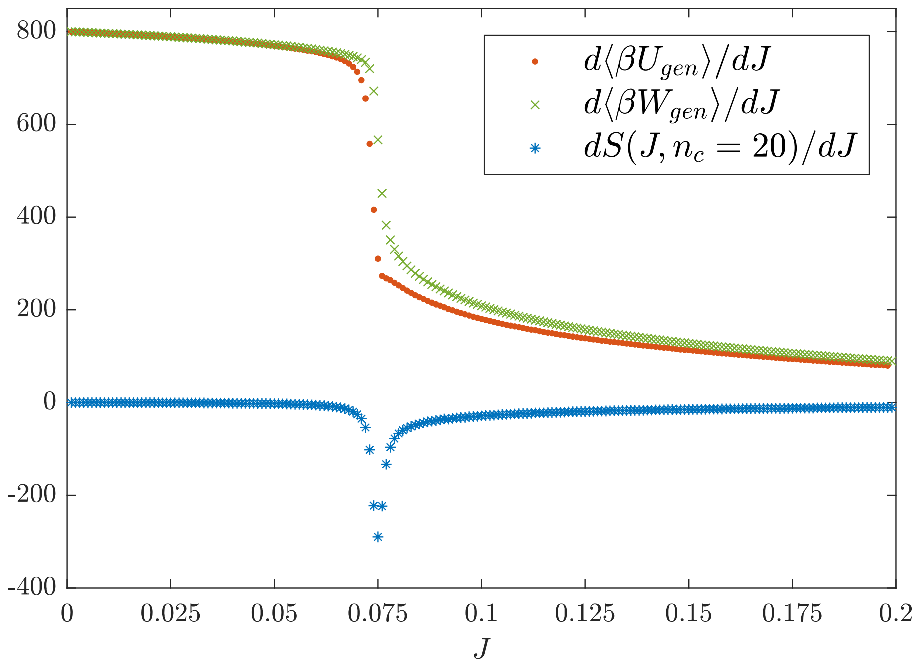


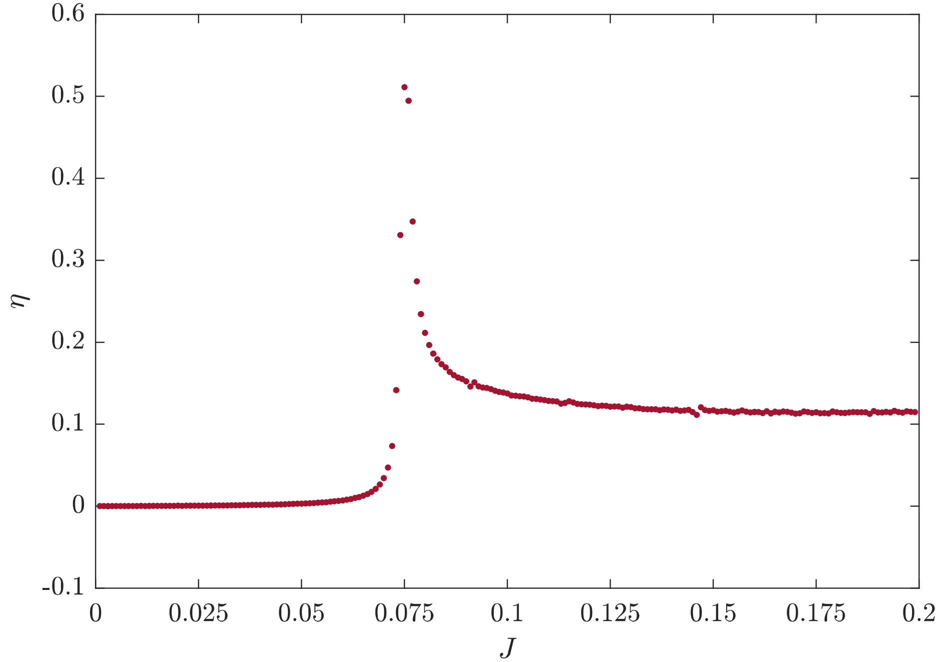




Thermodynamic efficiency of swarming behaviour

$$\eta = \frac{-dS(J, n_c)/dJ}{d\langle \beta W_{gen} \rangle / dJ}$$





E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, *Physical Review E,* 97, 012120, 2018.

2. Urban dynamics: MaxEnt + Lotka Volterra

$$H(\mathcal{Y}_{ij}) = -\sum_i \sum_j \mathcal{Y}_{ij} \log \mathcal{Y}_{ij},$$

constraints:

$$\sum_j \mathcal{Y}_{ij} = Y_i^{\text{out}}$$

$$\sum_i \sum_j \mathcal{Y}_{ij} A_j = A^{\text{tot}}$$

$$\sum_i \sum_j \mathcal{Y}_{ij} C_{ij} = C^{\text{tot}}$$

services income rent population

$$\frac{\mathrm{d}S_j}{\mathrm{d}t} = \epsilon(\mathcal{Y}_j^{\text{in}} - R_j P_j - K S_j)$$

$$\mathcal{Y}_{ij} = \frac{Y_i^* e^{\alpha A_j - \gamma C_{ij}}}{Z_i}$$

control parameters:

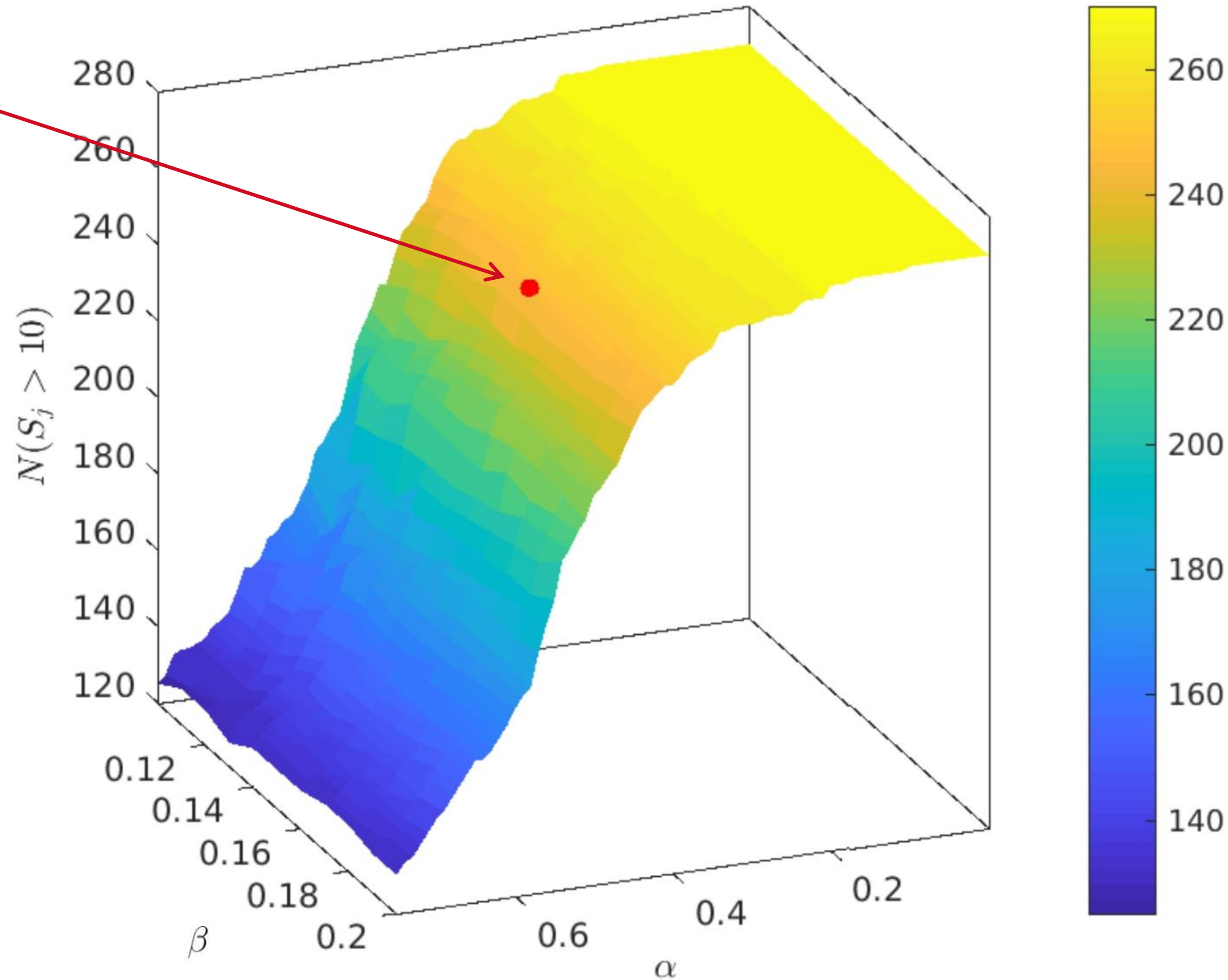
- social disposition α
- impedance to travel γ

order parameter: number of large suburbs

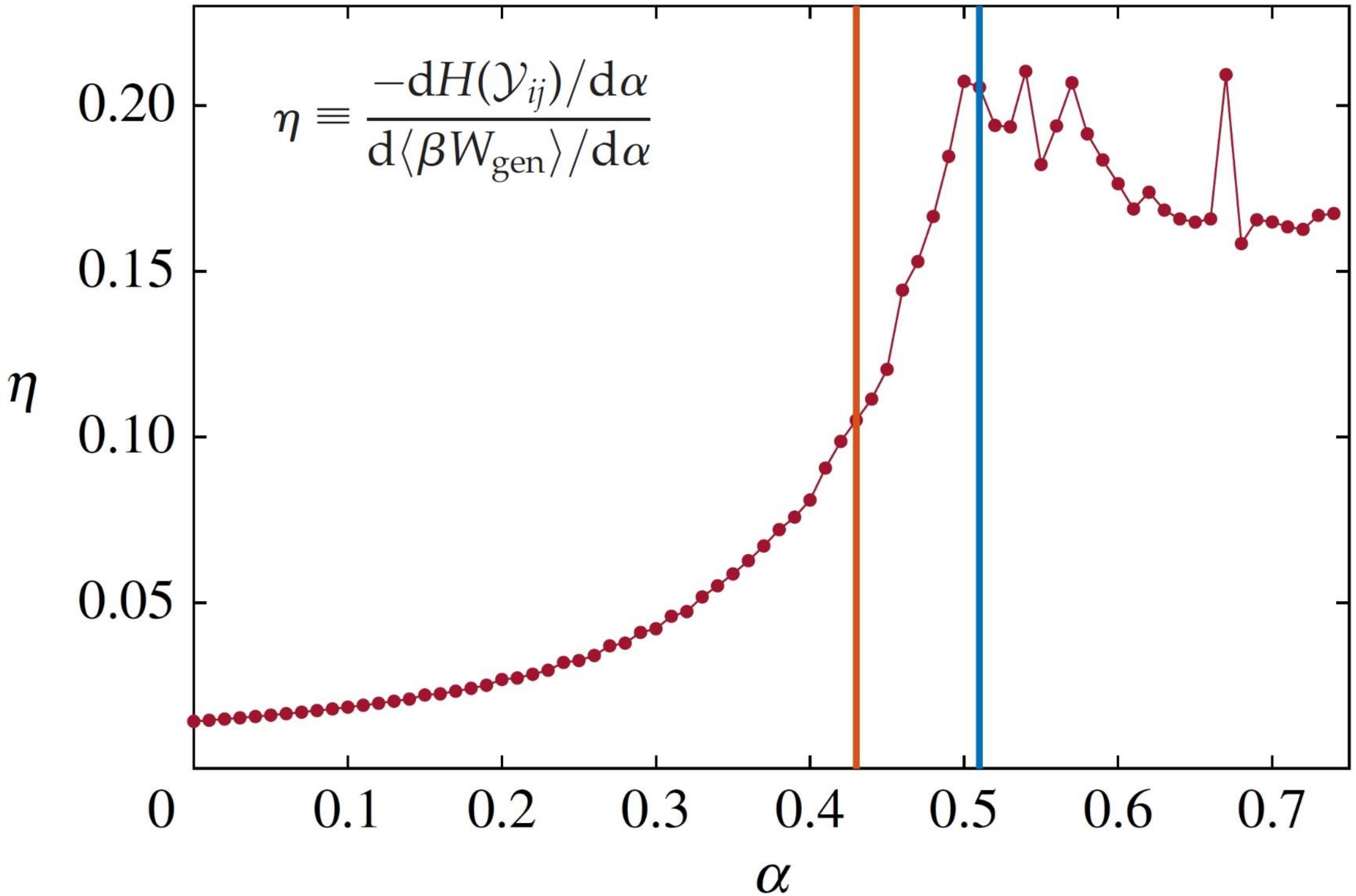
E. Crosato, R. Nigmatullin, M. Prokopenko, On critical dynamics and thermodynamic efficiency of urban transformations, *Royal Society Open Science*, 5: 180863, 2018.

Phase transition in the number of large suburbs

Sydney, 2011-2016



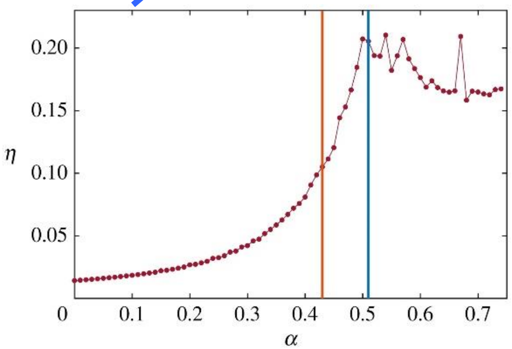
Greater Sydney: thermodynamic efficiency?

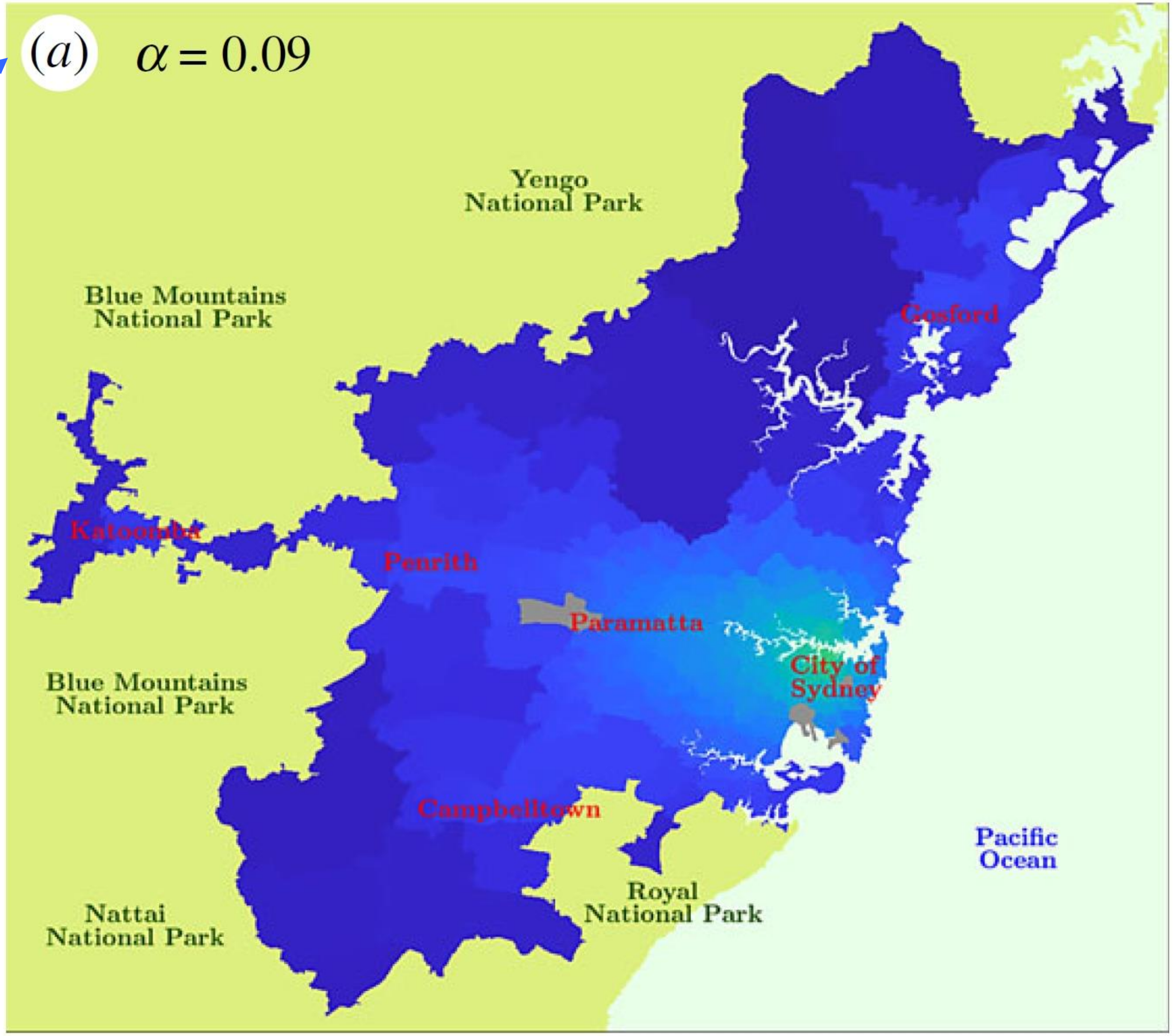


E. Crosato, R. Nigmatullin, M. Prokopenko, On critical dynamics and thermodynamic efficiency of urban transformations, *Royal Society Open Science*, 5: 180863, 2018.

Greater Sydney: monocentric / sprawling

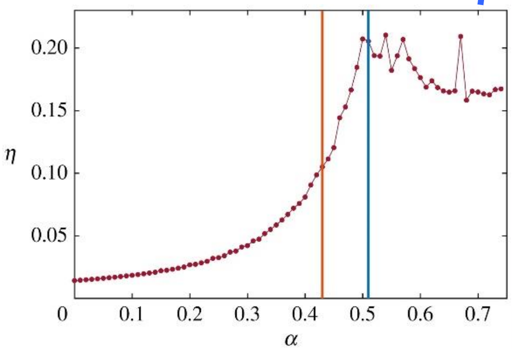
Low efficiency

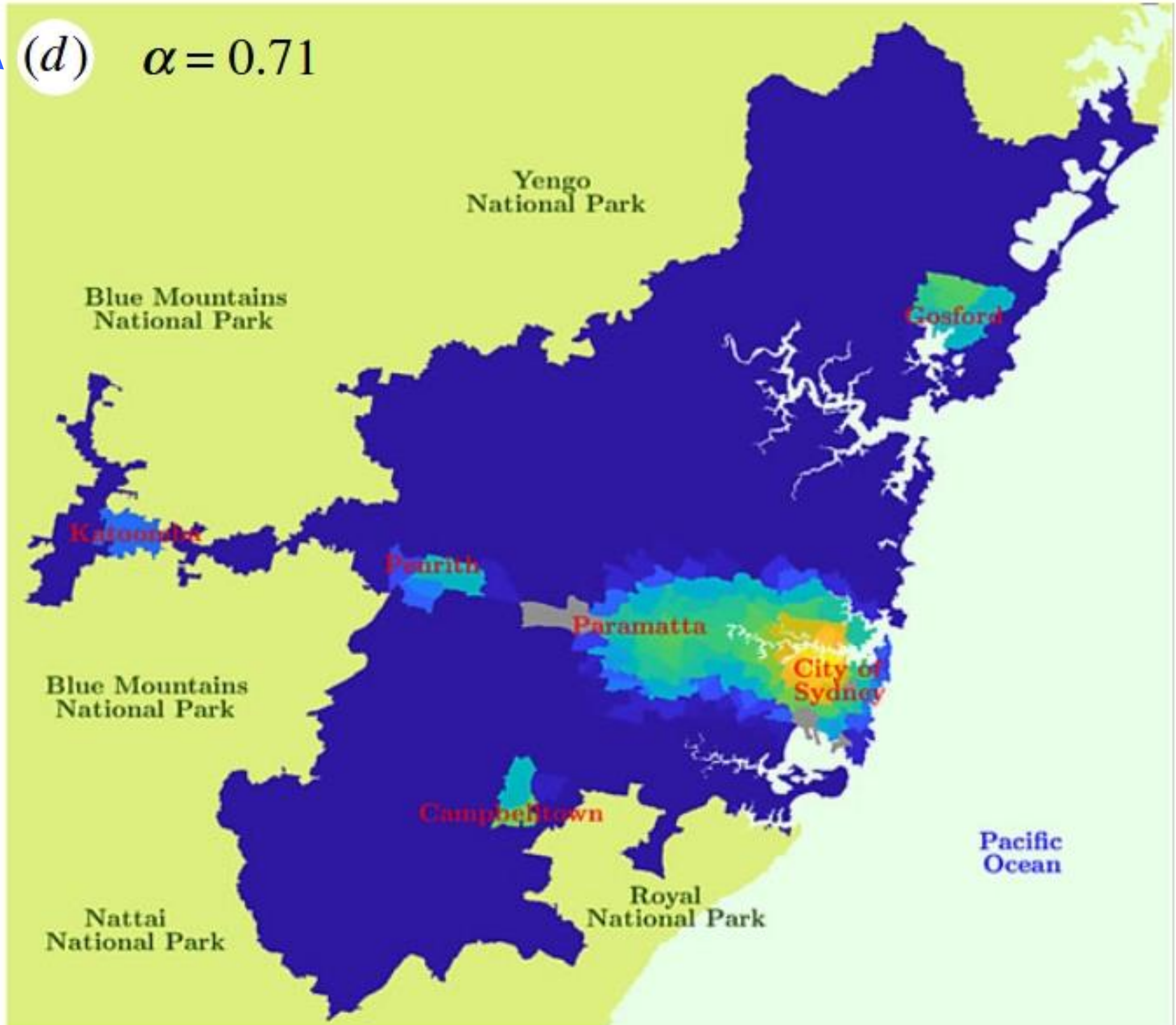




Greater Sydney: polycentric

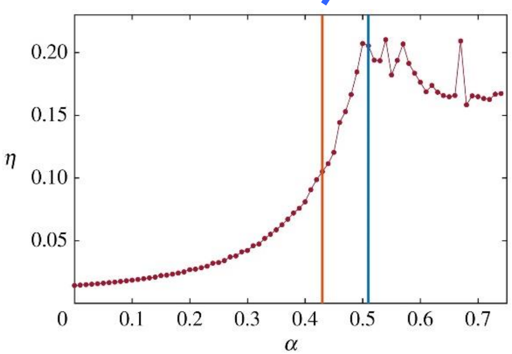
Medium to high efficiency

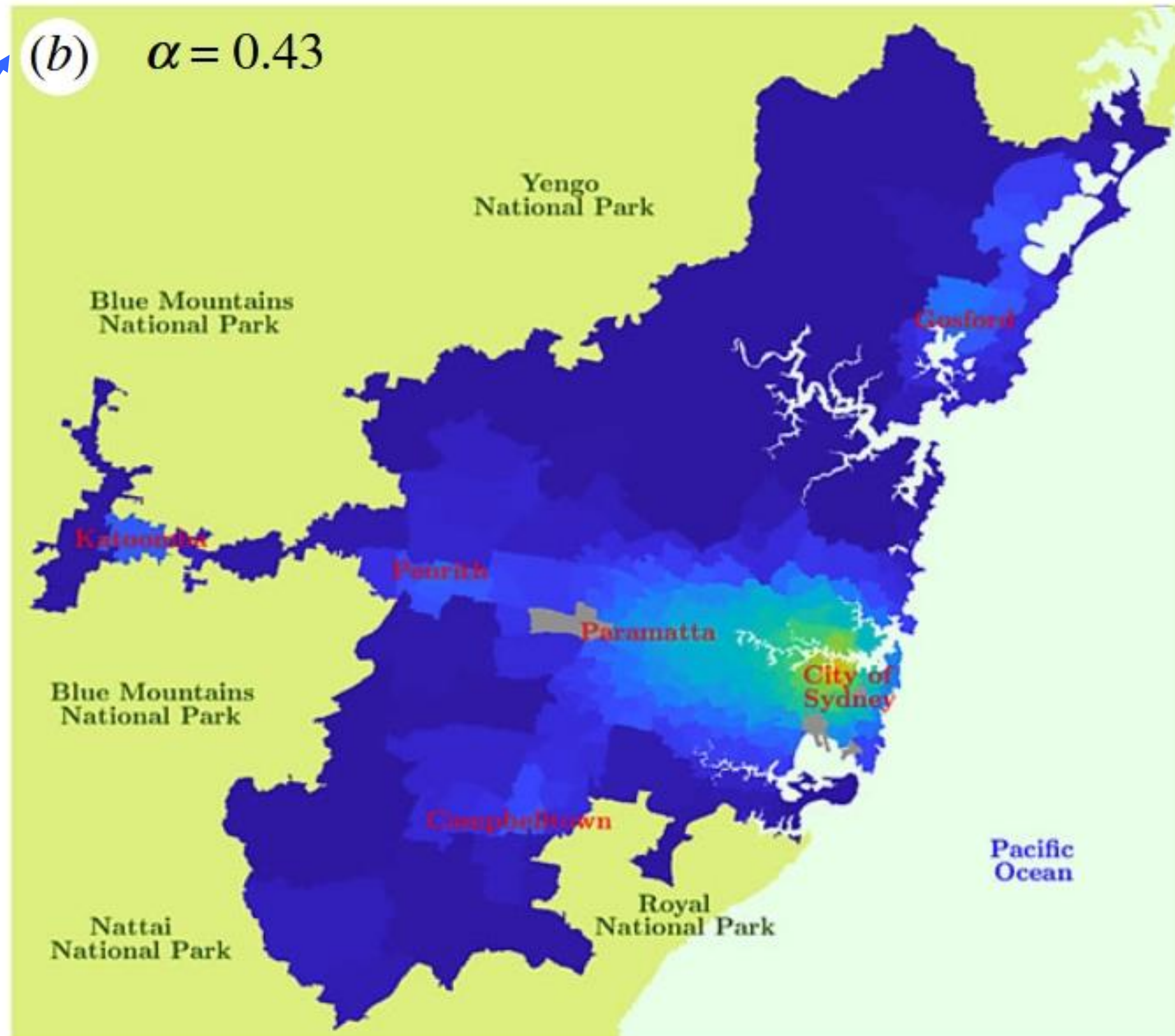




Greater Sydney: (2011-2016)

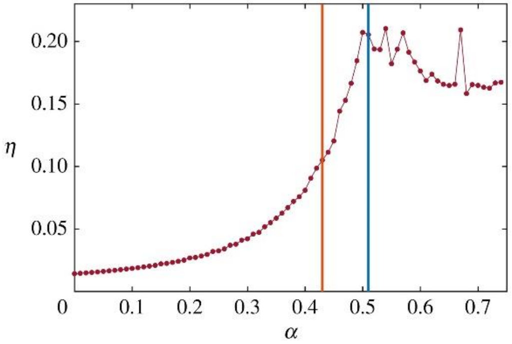
Medium efficiency

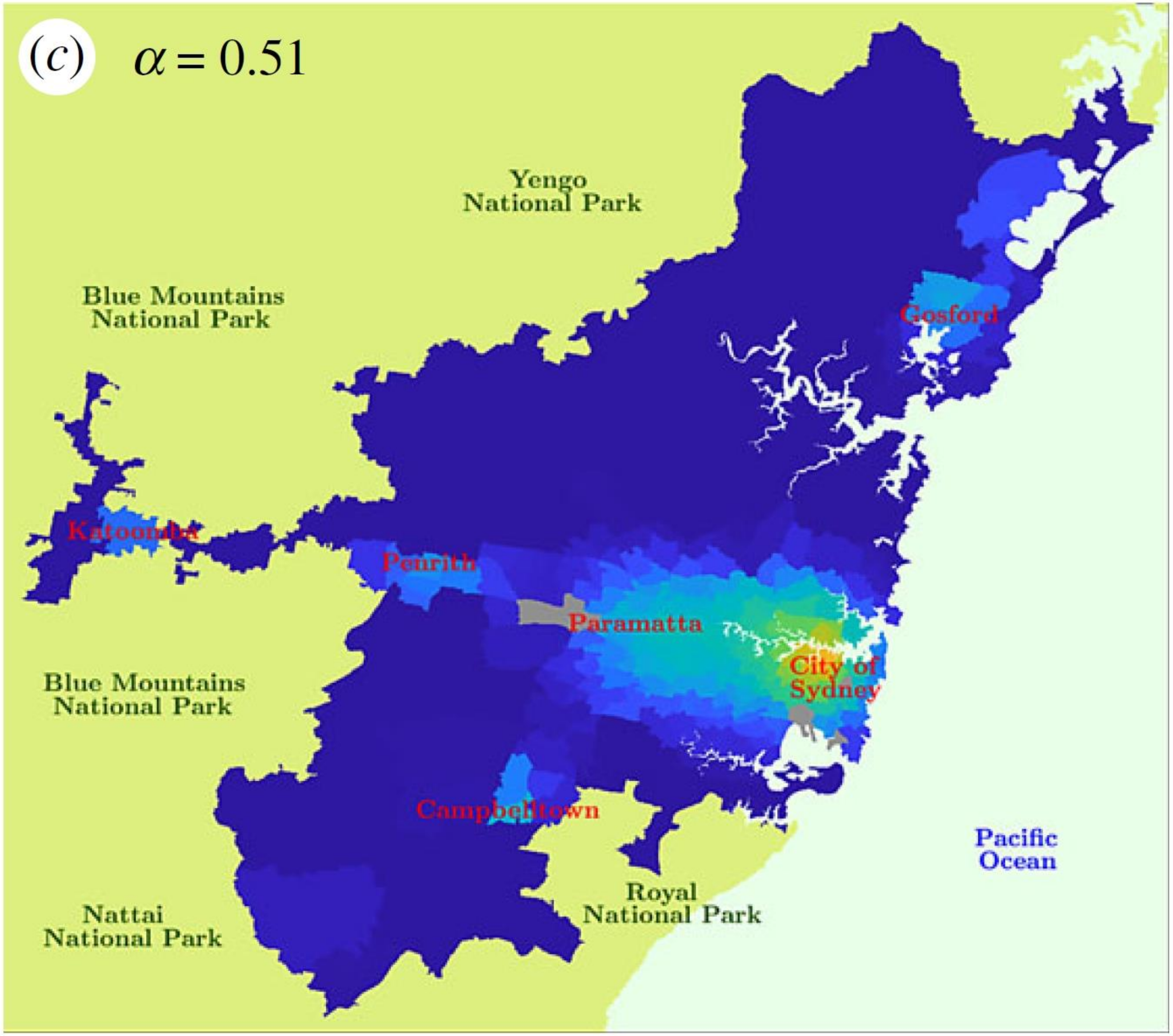




Greater Sydney: possible (critical regime)

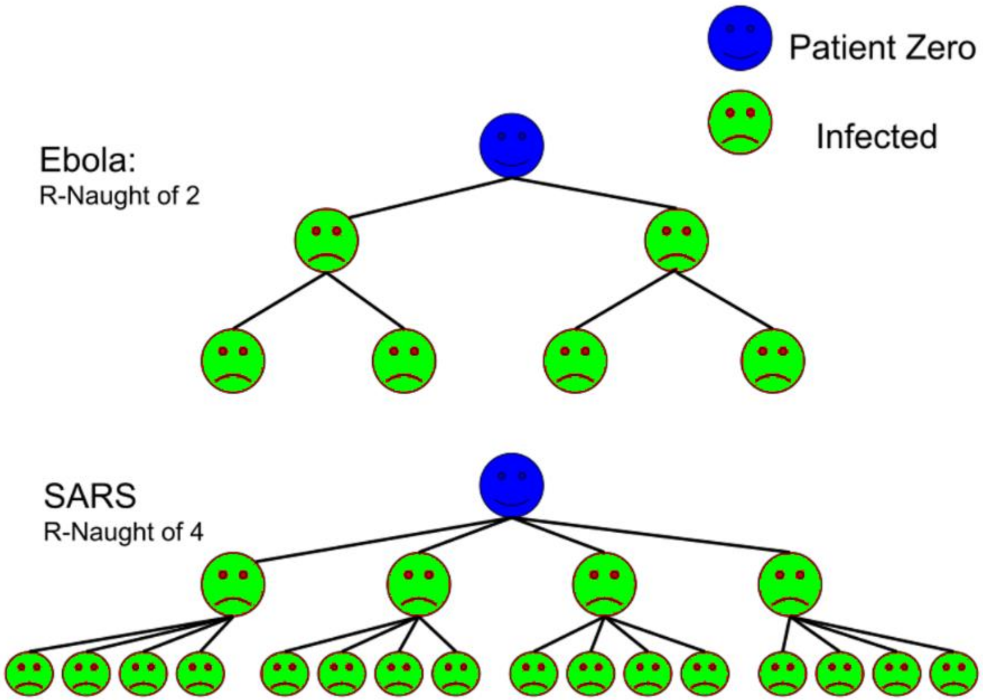
Higher efficiency





3. Epidemic dynamics

$$\left.\begin{aligned}\frac{\mathrm{d}S}{\mathrm{d}t} &= \gamma I - \beta IS \\ \frac{\mathrm{d}I}{\mathrm{d}t} &= \beta IS - \gamma I,\end{aligned}\right\} \beta/\gamma = R_0$$



Reproduction ratio

3. Epidemic dynamics (on networks)

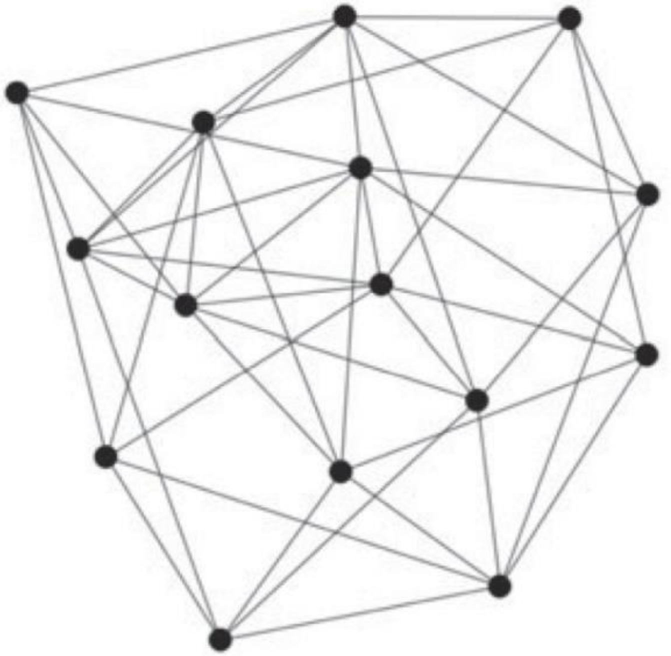
$$\frac{\mathrm{d}S}{\mathrm{d}t} = \gamma I - \beta I S$$

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \beta I S - \gamma I,$$

$$\beta/\gamma = R_0$$

$$\nu \qquad \delta$$

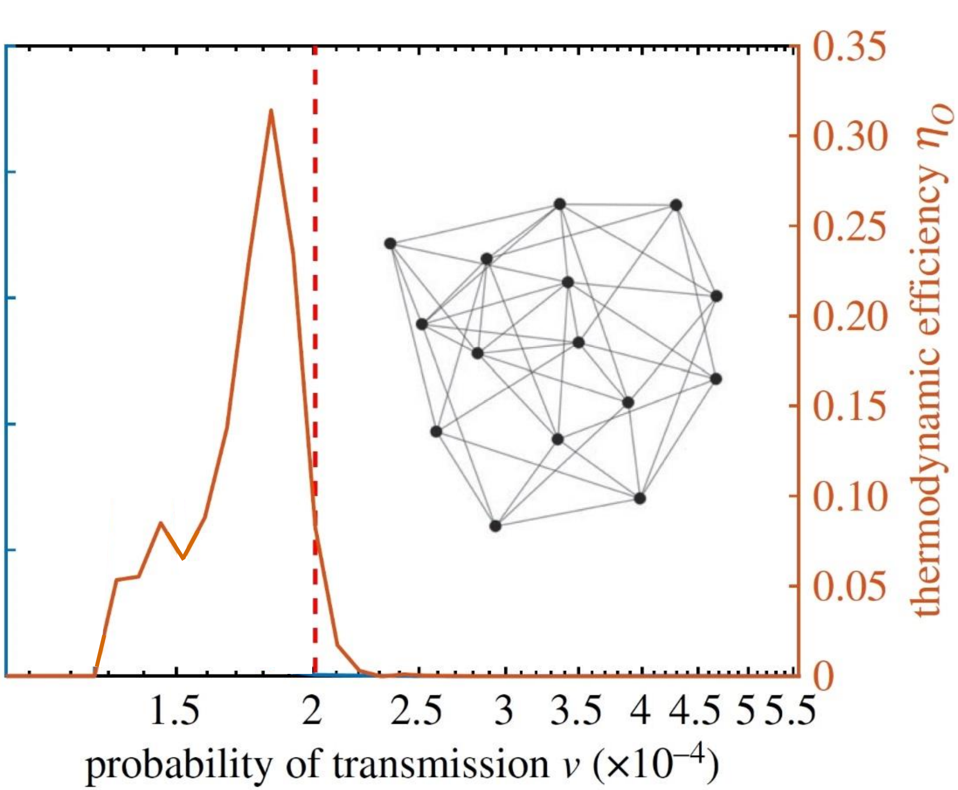
$$P(x_i) = 1 - (1 - \nu)^r$$

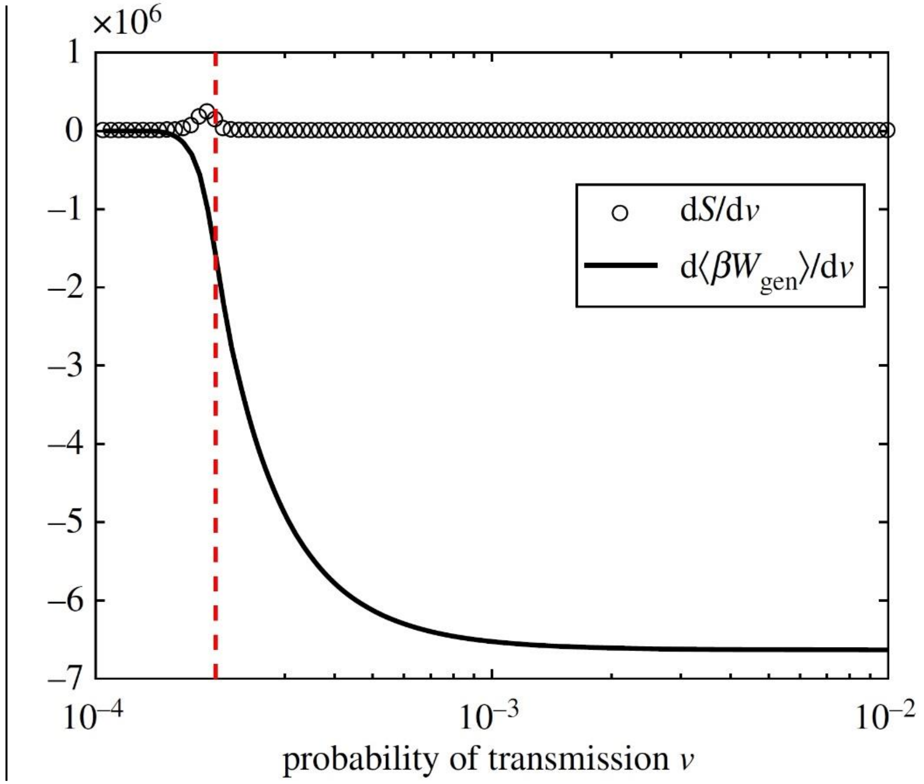


Reproduction ratio $R_0 = \dfrac{k\nu}{\nu + \delta - \nu\delta}$

N. Harding, R. Nigmatullin, M. Prokopenko, Thermodynamic efficiency of contagions: a statistical mechanical analysis of the SIS epidemic model, *Interface Focus*, 8 20180036, 2018.

Epidemics as thermodynamic phenomena

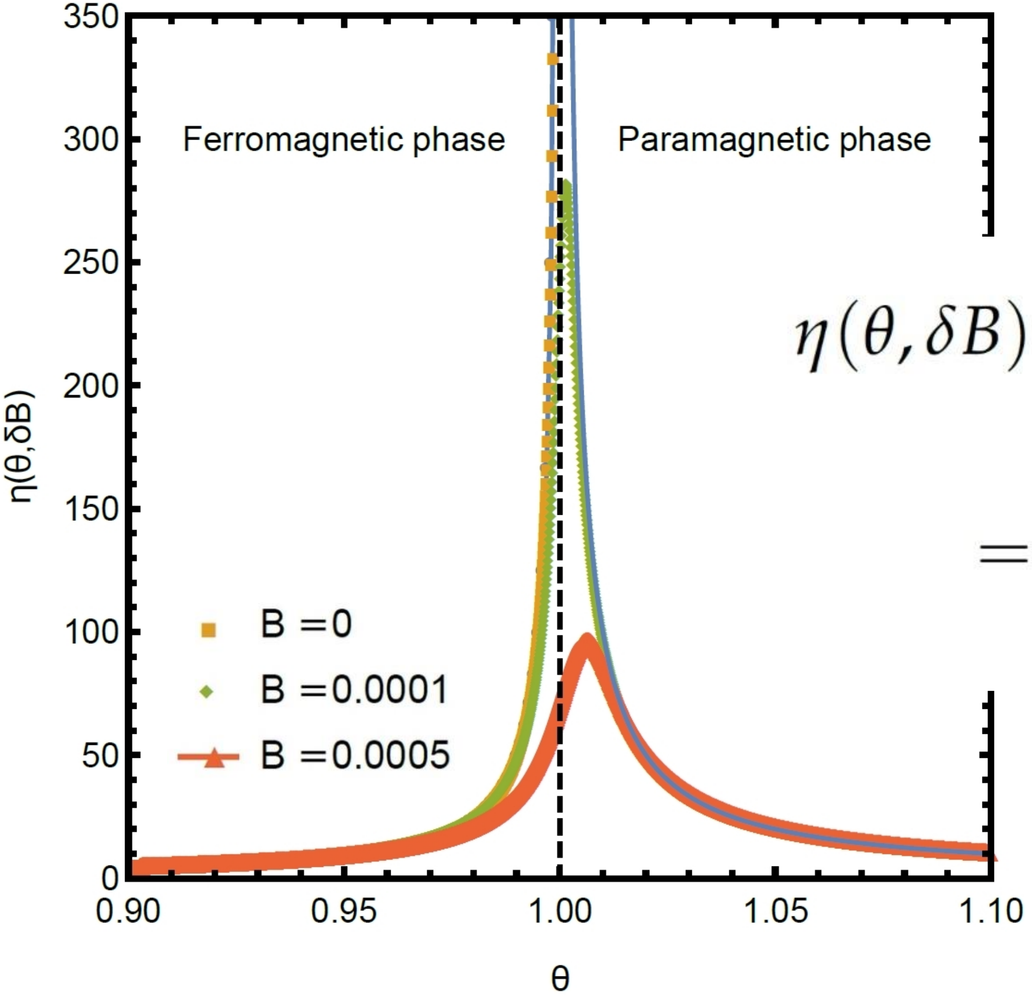




- intervention: reducing the transmission probability, expending the work
- pathogen emergence: increasing the transmission probability, extracting the work

N. Harding, R. Nigmatullin, M. Prokopenko, Thermodynamic efficiency of contagions: a statistical mechanical analysis of the SIS epidemic model, *Interface Focus*, 8 20180036, 2018.

4. Back to magnets…



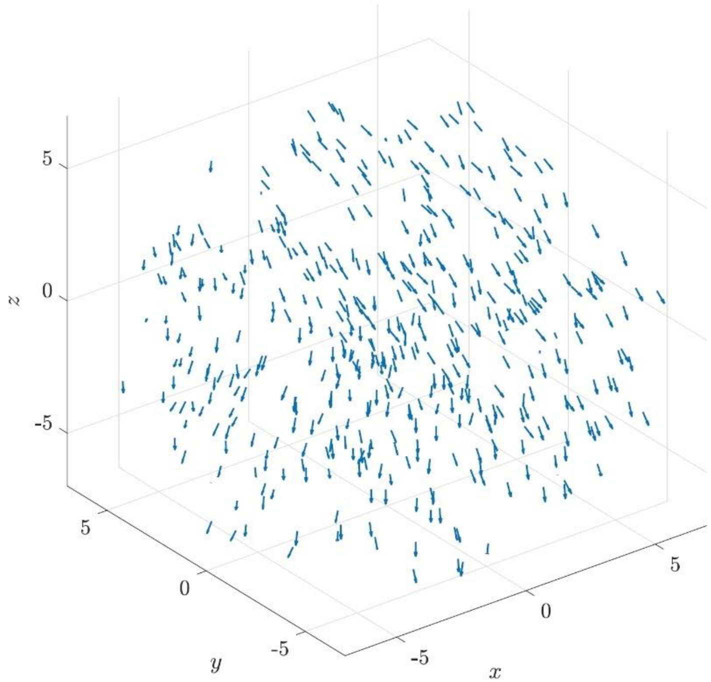
$$t \equiv (\theta - \theta_c)/\theta_c$$

$$\eta(\theta, \delta B) = \frac{1}{k_B} \frac{\partial s}{\partial B} \Big/ \frac{\partial f}{\partial B}$$

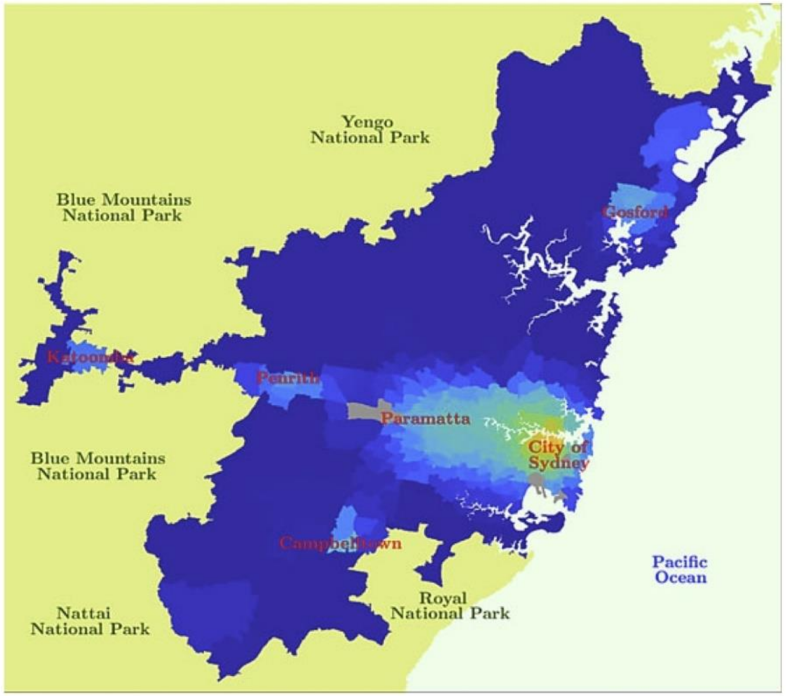
$$= \begin{cases} -\frac{1}{k_B} \frac{1}{2} t^{-1} & \text{for } t < 0, \\ \frac{1}{k_B \theta_c} t^{-1} & \text{for } t > 0. \end{cases}$$

R. Nigmatullin, M. Prokopenko, Thermodynamic efficiency of interactions in self-organizing systems, *Entropy*, 23(6): 757, 2021.

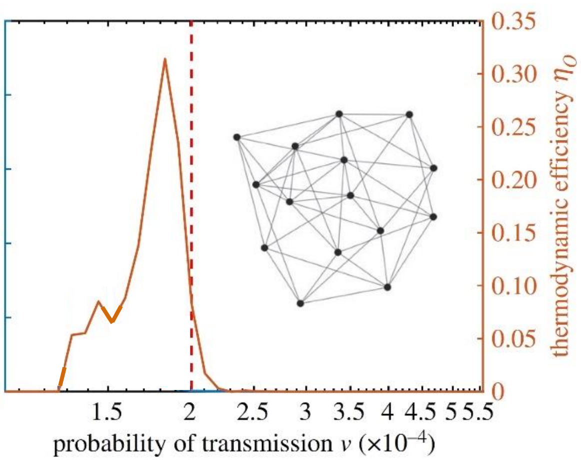
Summary (1/2)



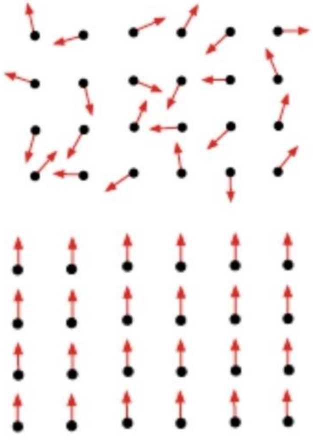
5
0
-5
z
5
0
-5
y
-5
0
5
x

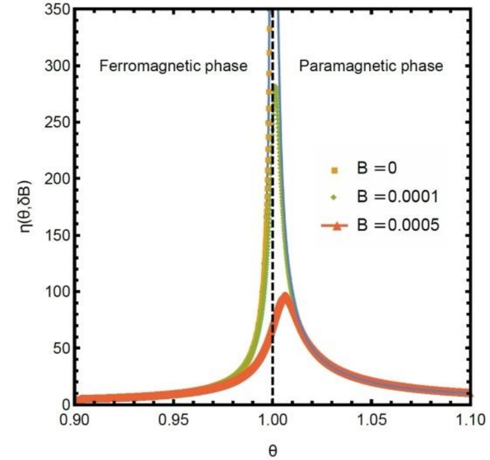


Yengo
National Park
Blue Mountains
National Park
Gosford
Katoomba
Penrith
Paramatta
City of
Sydney
Blue Mountains
National Park
Campbelltown
Pacific
Ocean
Royal
National Park
Nattai
National Park



0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
thermodynamic efficiency η_O
1.5 2 2.5 3 3.5 4 4.5 5 5.5
probability of transmission ν ($\times 10^{-4}$)





Ferromagnetic phase
Paramagnetic phase
B =0
B =0.0001
B =0.0005
η(θ,δB)
350
300
250
200
150
100
50
0
0.90 0.95 1.00 1.05 1.10
θ

Summary (2/2)

- Critical regime: balance between order and chaos
- Thermodynamic and computational perspectives:
  - rate of work carried out to change control parameter = accumulated sensitivity of distributed computation (integral of Fisher information)
- Thermodynamic efficiency:
  - the reduction in uncertainty (the increase in order) from an expenditure of work for a given value of control parameter
  - diverges at critical point for model systems (e.g., Ising model)

$$\eta(X, \delta X) = -\frac{1}{k_B} \frac{\beta}{|T - T_c|}$$

- *Principle of Super-efficiency*: efficiency of self-organisation is maximal at critical points in dynamical systems

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