

Pattern formation and critical regimes during social and epidemic dynamics

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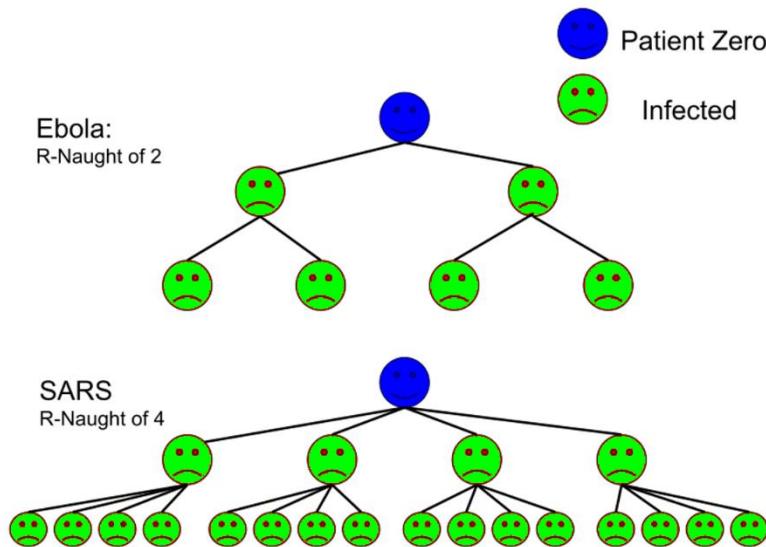
Sydney Institute for Infectious Diseases



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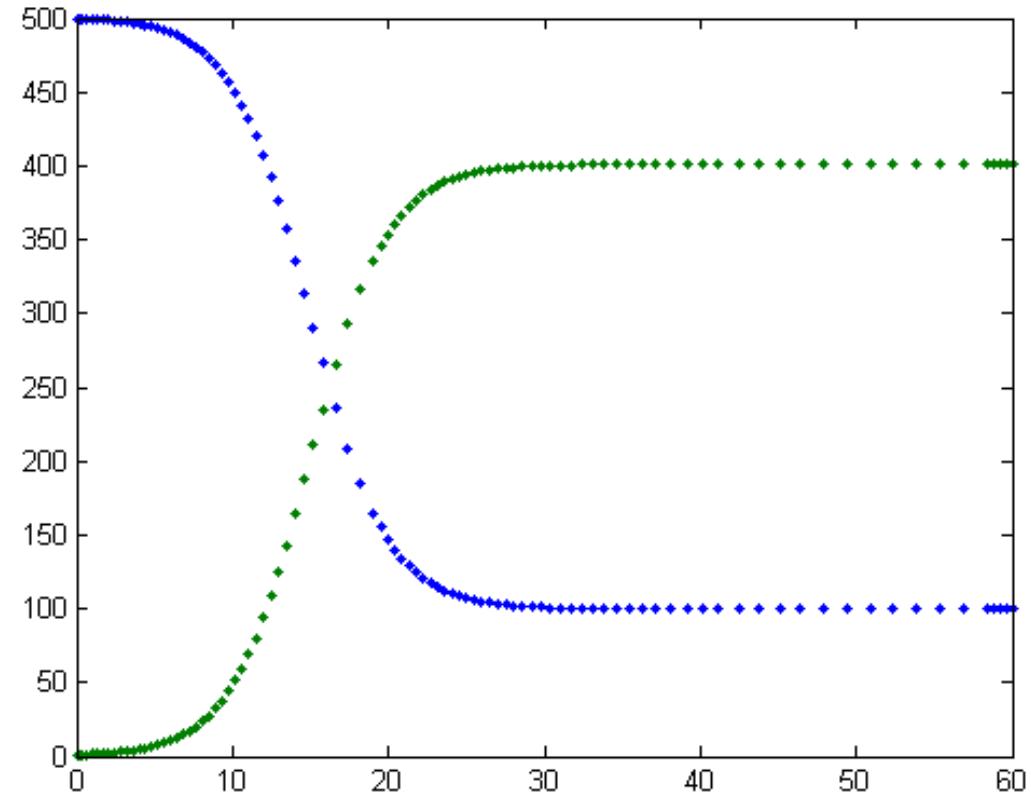
- Epidemic modelling: preliminaries
- Pattern formation during social dynamics
- Phase transitions and Fisher information
- Thermodynamic efficiency of interactions
 - Curie-Weiss model
 - Epidemics on networks

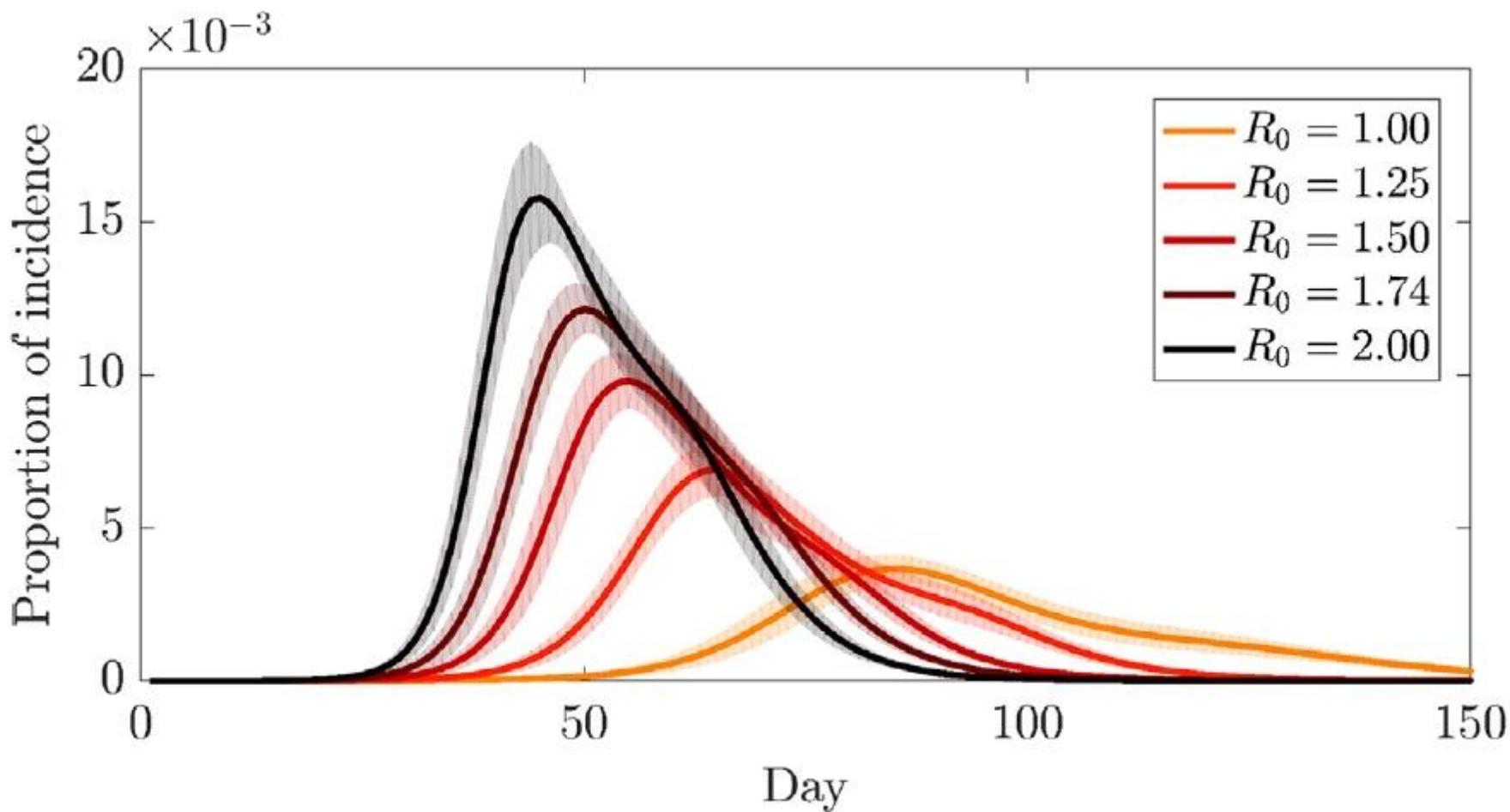


$$\frac{dS}{dt} = \gamma I - \beta IS$$
$$\frac{dI}{dt} = \beta IS - \gamma I,$$

$\beta / \gamma = R_0$

SIS: Susceptible - Infectious - Susceptible

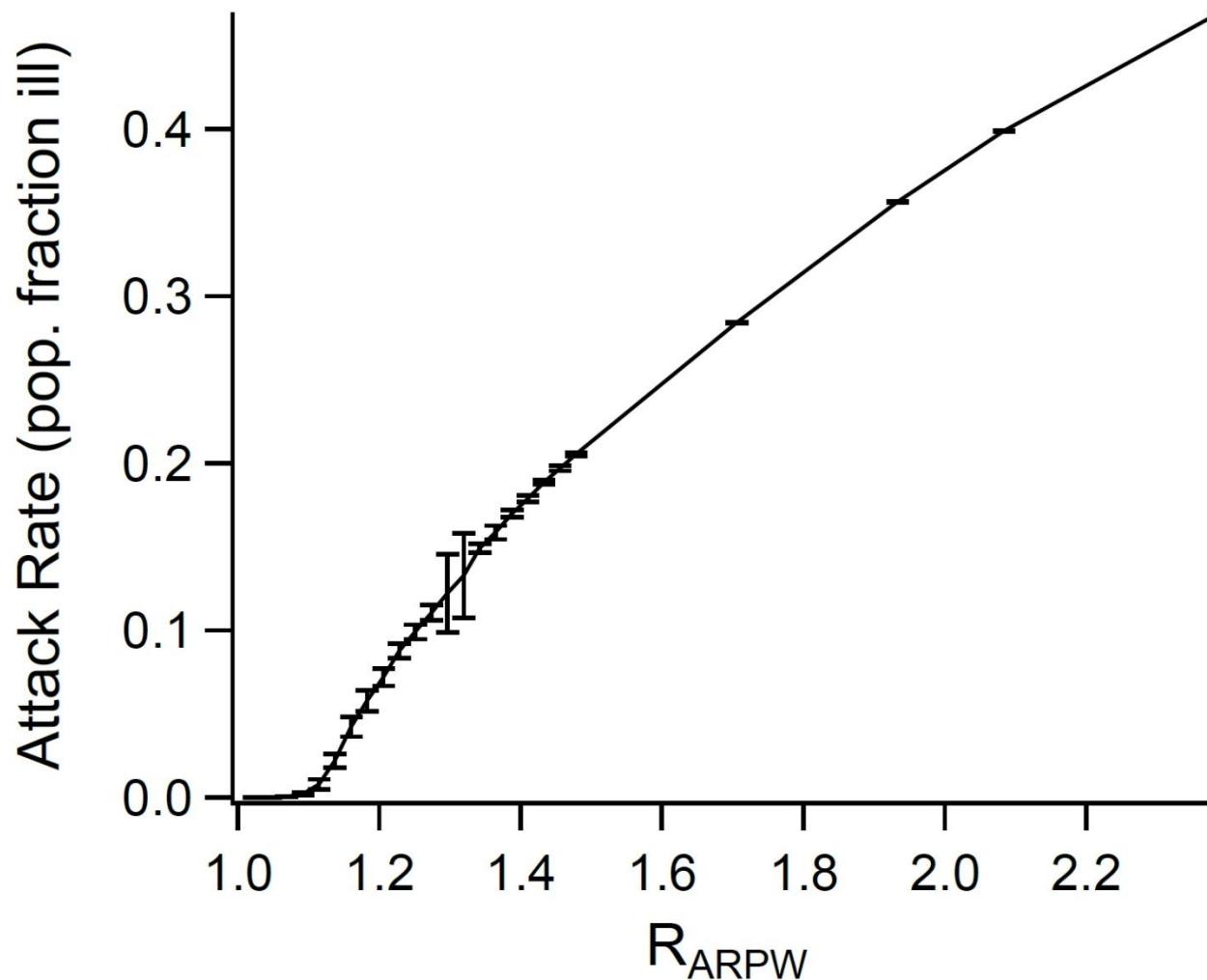


Epidemic modelling: reproductive ratio R_0 

O. M. Cliff, N. Harding, M. Piraveenan, E. Y. Erten, M. Gambhir, M. Prokopenko, Investigating Spatiotemporal Dynamics and Synchrony of Influenza Epidemics in Australia: An Agent-Based Modelling Approach, *Simulation Modelling Practice and Theory*, 87, 412-431, 2018.

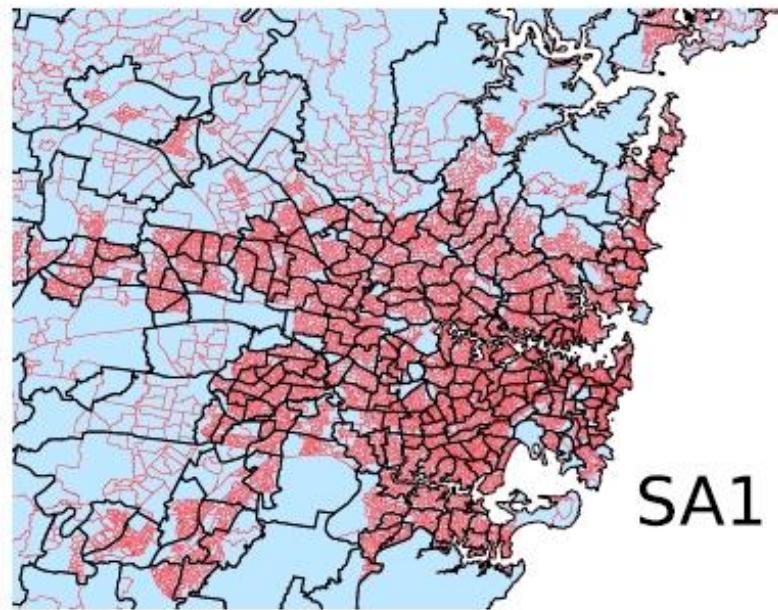


Epidemic threshold: reproductive number R_0



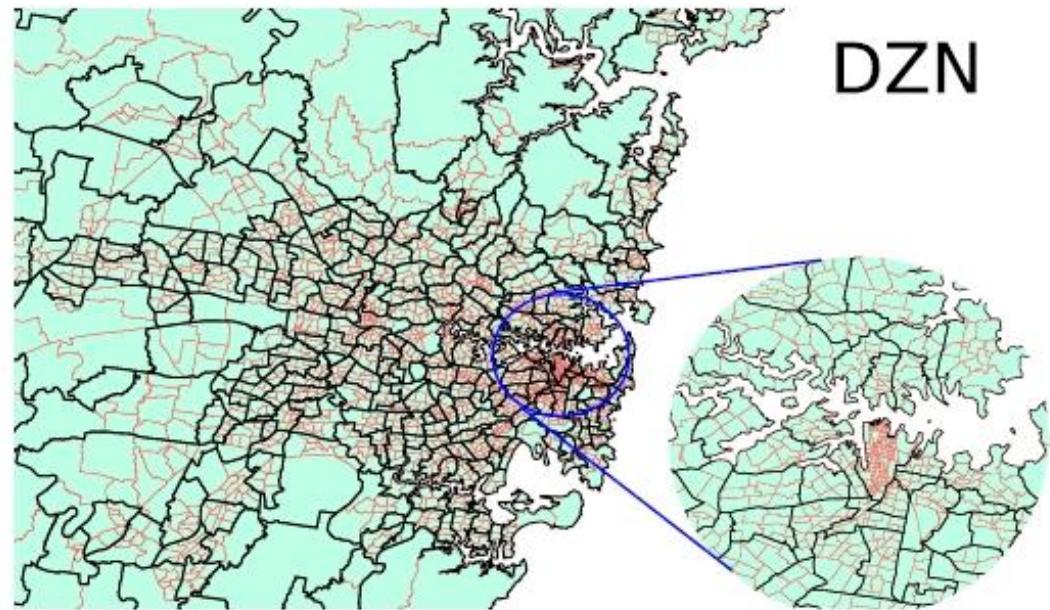
Population partitions: residential areas and destination zones

a)



SA1

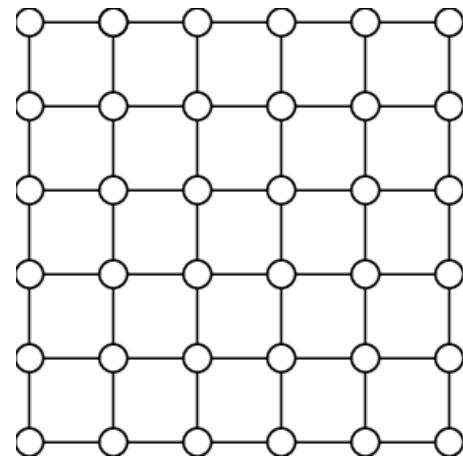
b)



DZN

Fig. 1 Maps of the Greater Sydney region illustrating the distribution of population partitions. (a) A map of the Greater Sydney region showing SA2 (black) and SA1 (red) population partitions. (b) A map of the same area showing SA2 (black) and DZN (red) partitions. The inset in (b) zooms in on the Sydney central business district to illustrate the much denser packing of DZN partitions in that area.

Dynamic meta-population epidemic (SIS) model



$$\frac{dI_i}{dt} = -\gamma I_i + \beta \sum_{j,k} \phi_{ij}^S(\mathbf{I}, \mathbf{C}) \phi_{kj}^I(\mathbf{I}, \mathbf{C}) \frac{S_i I_k}{\hat{N}_j(\mathbf{I}, \mathbf{C})}$$

$$\hat{N}_j(\mathbf{I}, \mathbf{C}) = \sum_k S_k \phi_{kj}^S(\mathbf{I}, \mathbf{C}) + I_k \phi_{kj}^I(\mathbf{I}, \mathbf{C})$$

Benefit

$$b_j = N_j^{-1}(N_j - I_j)$$

MaxEnt
Principle

$$H_Y = - \sum_y p_Y(y) \ln p_Y(y)$$

constraints

$$B^I = \sum_{i,j} I_i \phi_{ij}^I(\mathbf{I}, \mathbf{C}) b_j / \sum_i I_i$$

vs
control
parameters

$$B^S = \sum_{i,j} S_i \phi_{ij}^S(\mathbf{I}, \mathbf{C}) b_j / \sum_i S_i$$

$$C = \sum_{i,j} (I_i \phi_{ij}^I(\mathbf{I}, \mathbf{C}) + S_i \phi_{ij}^S(\mathbf{I}, \mathbf{C})) c_{ij} / \sum_i (I_i + S_i)$$

MaxEnt
Principle

$$H_Y = - \sum_y p_Y(y) \ln p_Y(y)$$

$$\begin{pmatrix} \alpha^S \\ \alpha^I \\ \omega \end{pmatrix}$$

Specify constraints: Infer model parameters

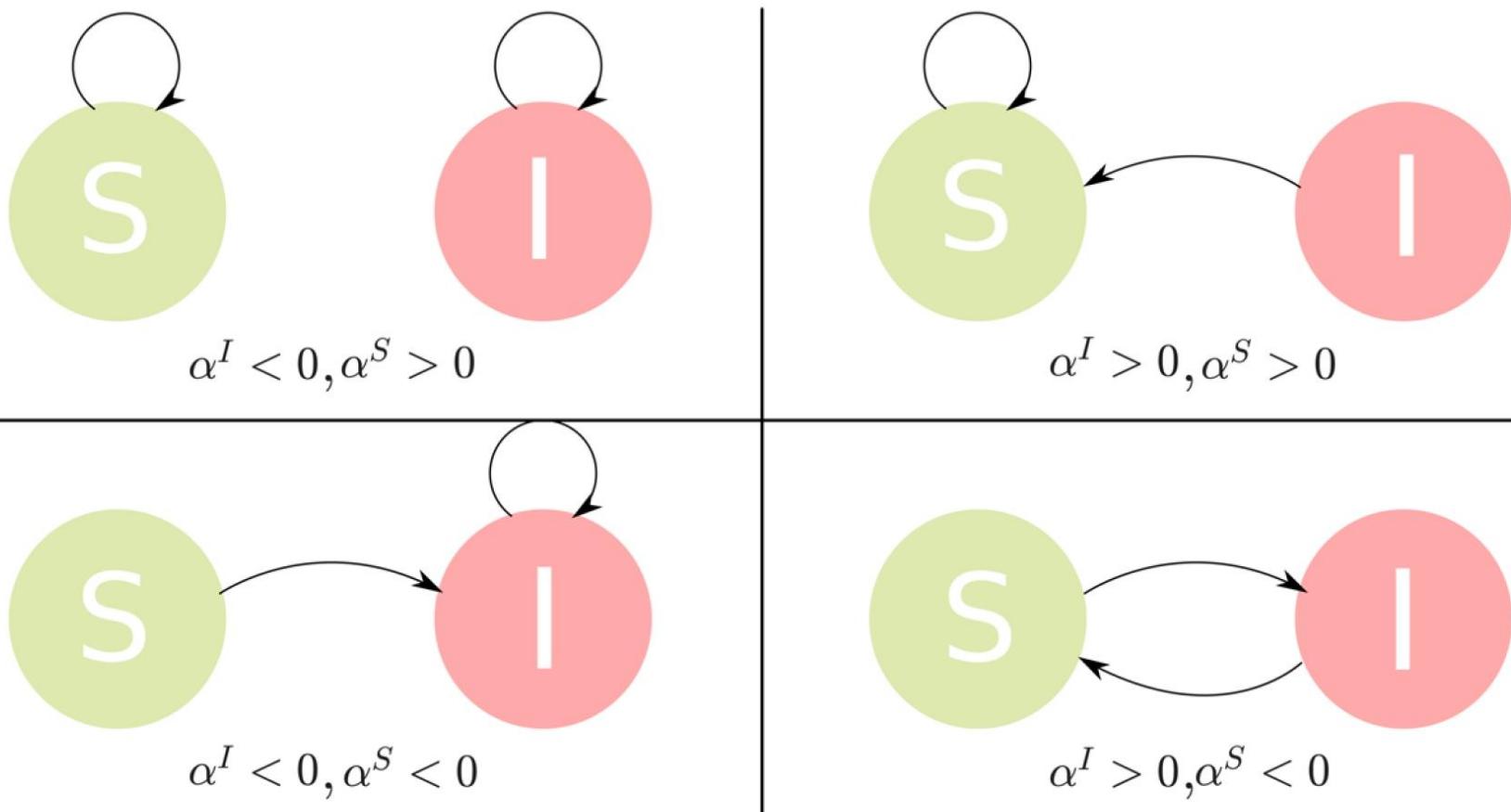
$$\leftrightharpoons$$

Specify parameters: Model generates quantities

$$\begin{pmatrix} B^S \\ B^I \\ C \end{pmatrix}$$

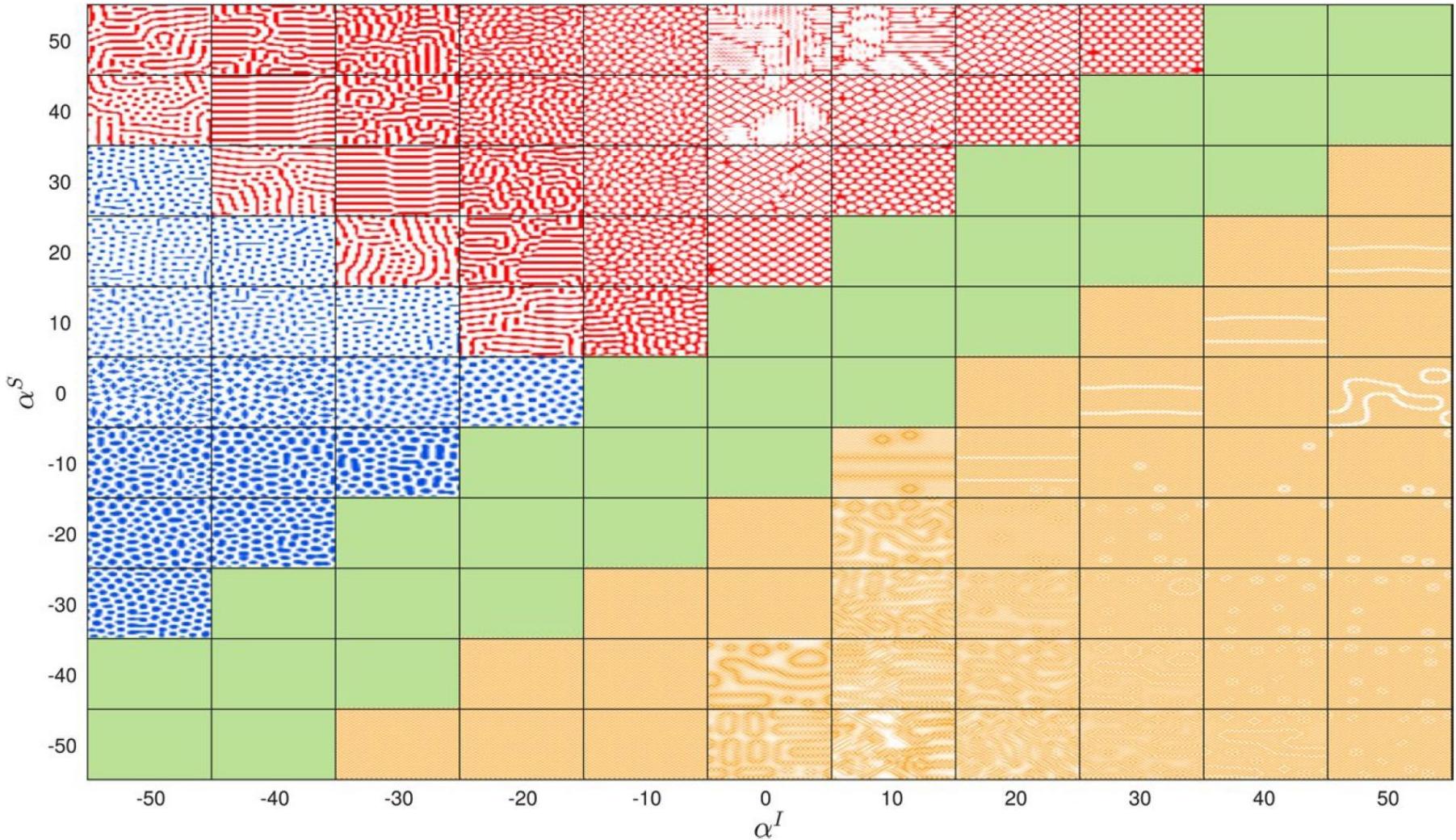
Bounded rationality: four types of dynamics

$$\phi_{ij}^x(\mathbf{I}, \mathbf{C} | \alpha^x, \omega) = Z_{x,i}^{-1} \exp\left(\alpha^x b_j - \omega c_{ij}\right)$$

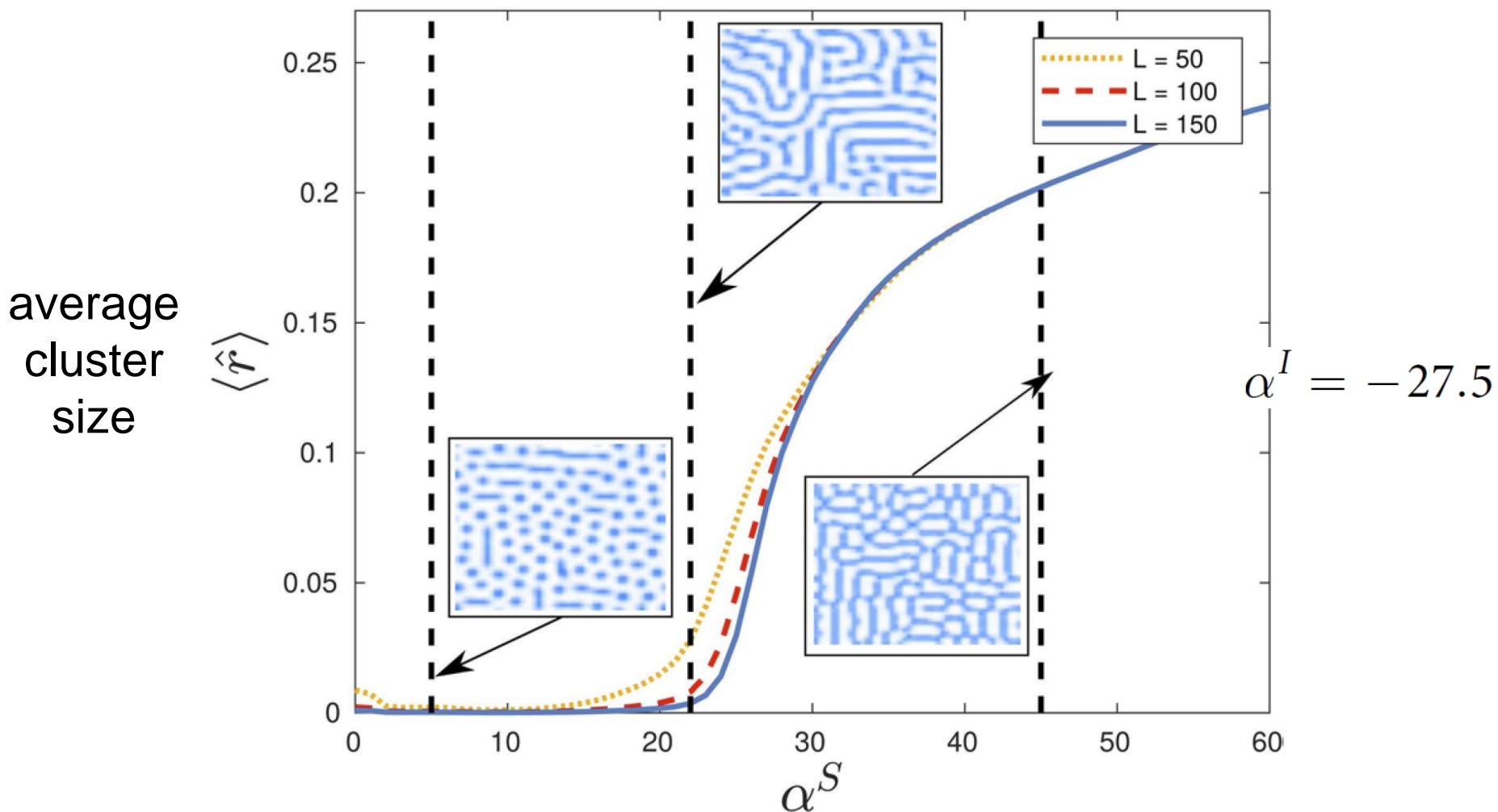


Behaviour-induced spatial morphology

$$\phi_{ij}^x(\mathbf{I}, \mathbf{C} | \alpha^x, \omega) = Z_{x,i}^{-1} \exp\left(\alpha^x b_j - \omega c_{ij}\right)$$

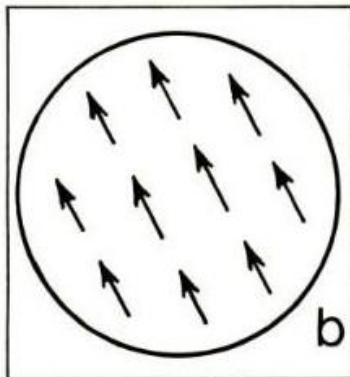


Spatial morphology: critical regimes

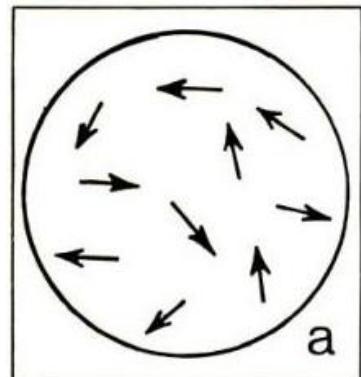




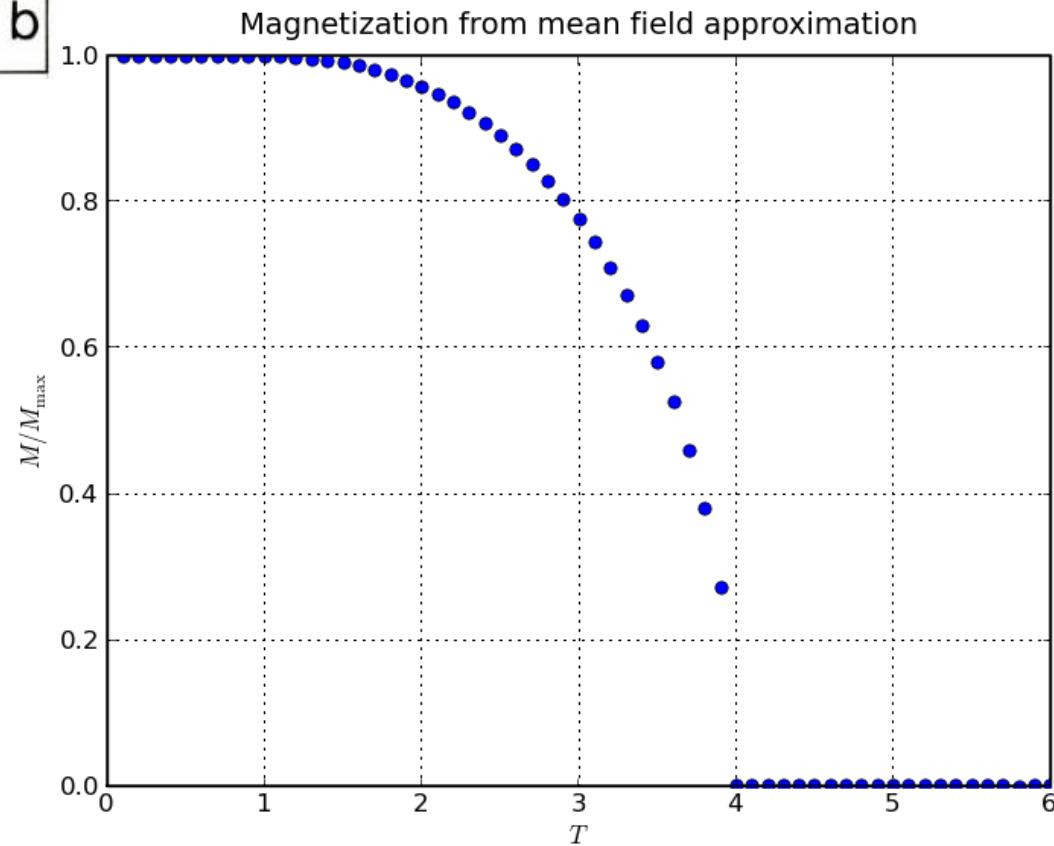
Phase transitions and order parameters



anisotropic
“coherent”

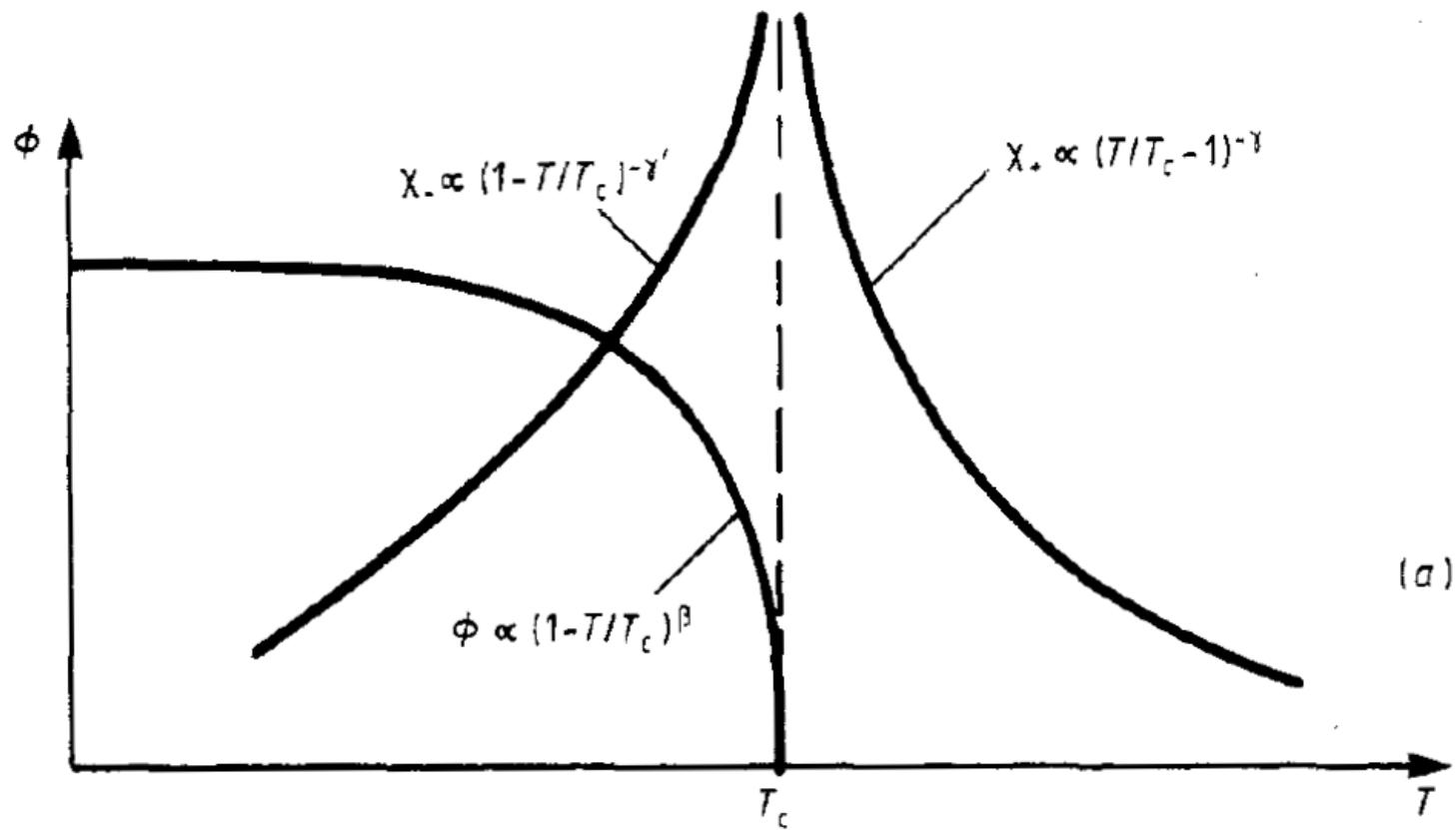


isotropic
“disordered”



Derivative of order parameter (divergence)

K Binder (1987)

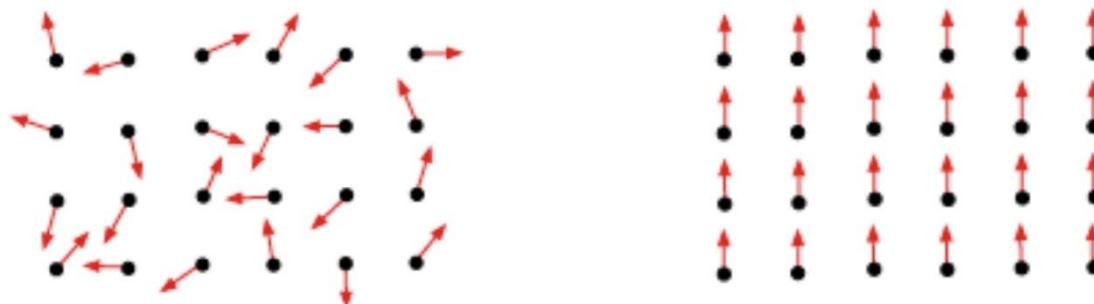


Fisher Information and sensitivity

A way of measuring the amount of information that an observable random variable X has about an unknown parameter θ



$$F(\theta) = \int_x \left(\frac{\partial \ln(p(x|\theta))}{\partial \theta} \right)^2 p(x|\theta) dx$$

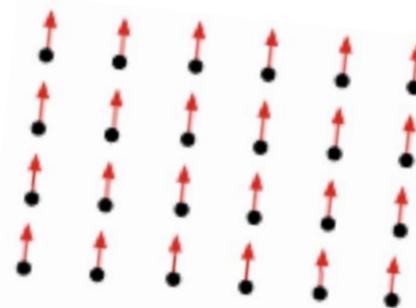
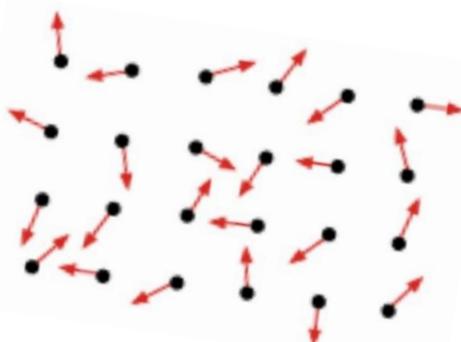


Fisher Information and sensitivity

A way of measuring the amount of information that an observable random variable X has about an unknown parameter θ



$$F(\theta) = \int_x \left(\frac{\partial \ln(p(x|\theta))}{\partial \theta} \right)^2 p(x|\theta) dx$$



$$G(T, \theta_i) = U(S, \phi_i) - TS - \phi_i \theta_i$$

$$F_{ij}(\theta) = \beta \frac{\partial \phi_i}{\partial \theta_j}$$

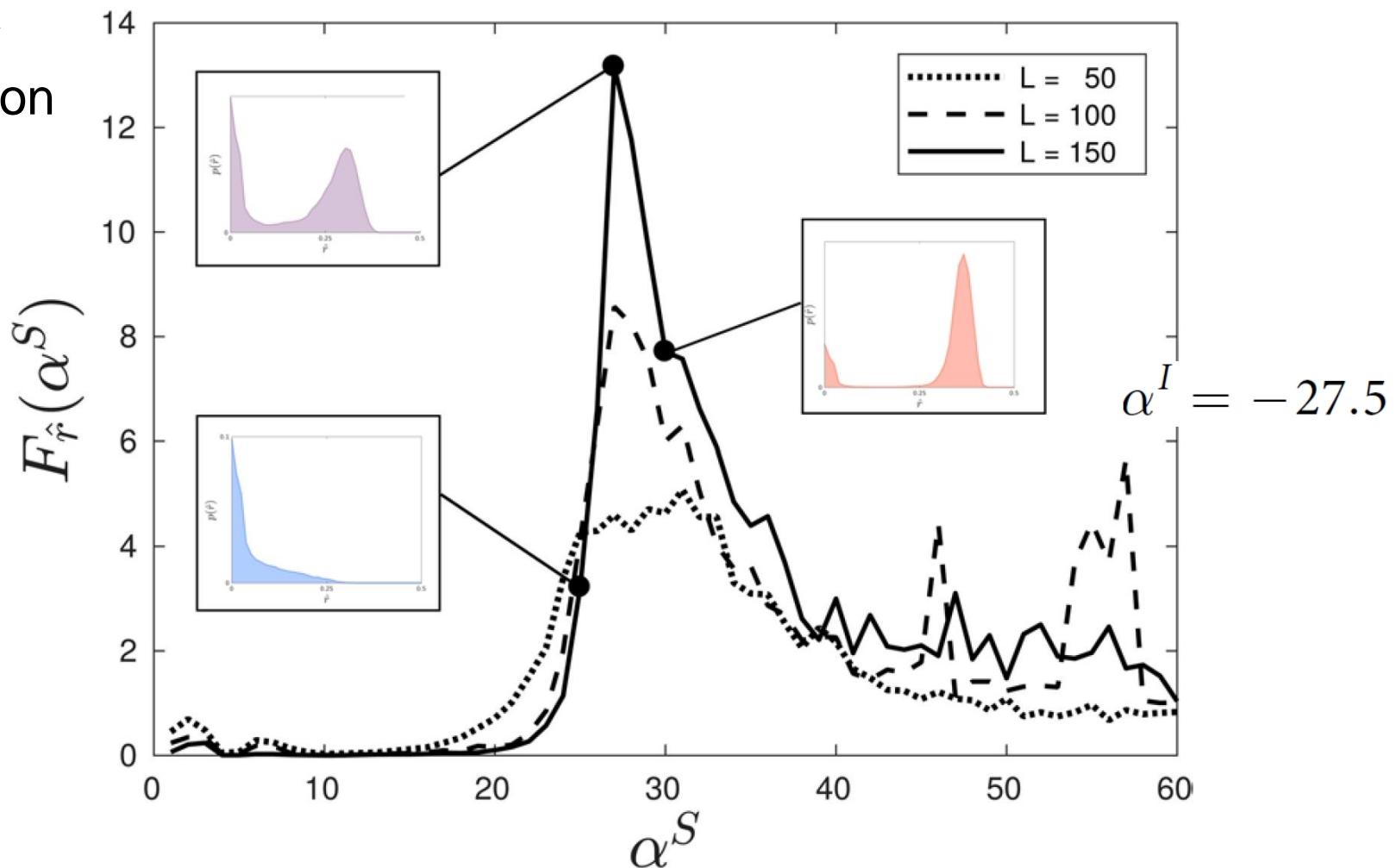
Fisher information matrix

Rate of change of the
order parameter

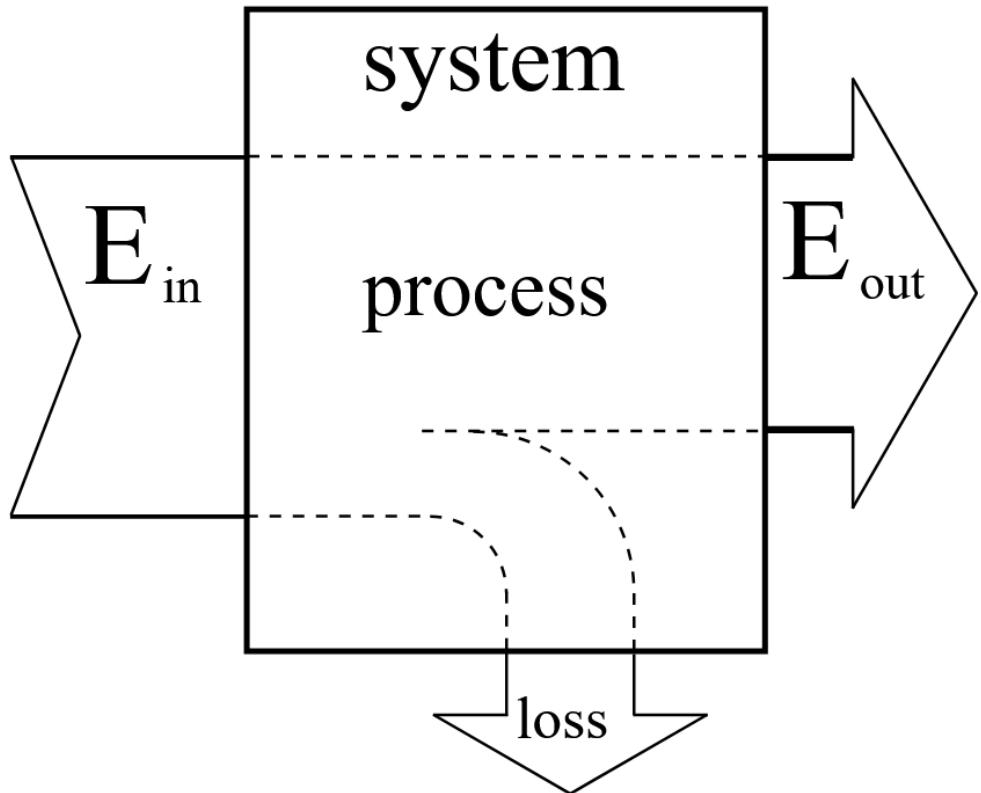


Spatial morphology: Fisher information

Fisher
information



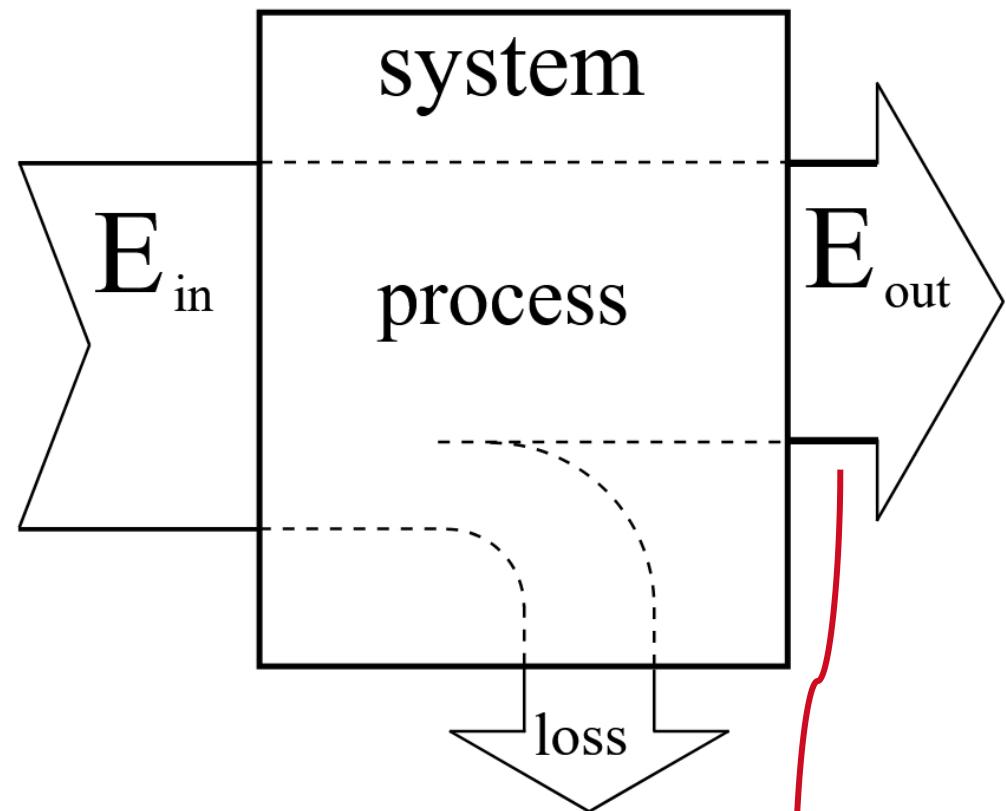
- bounded rationality induces transitions in morphology
(spots → labyrinth → gaps)
- Fisher information diverges at critical points



$$Q_{in} = |W_{out}| + |Q_{out}|$$

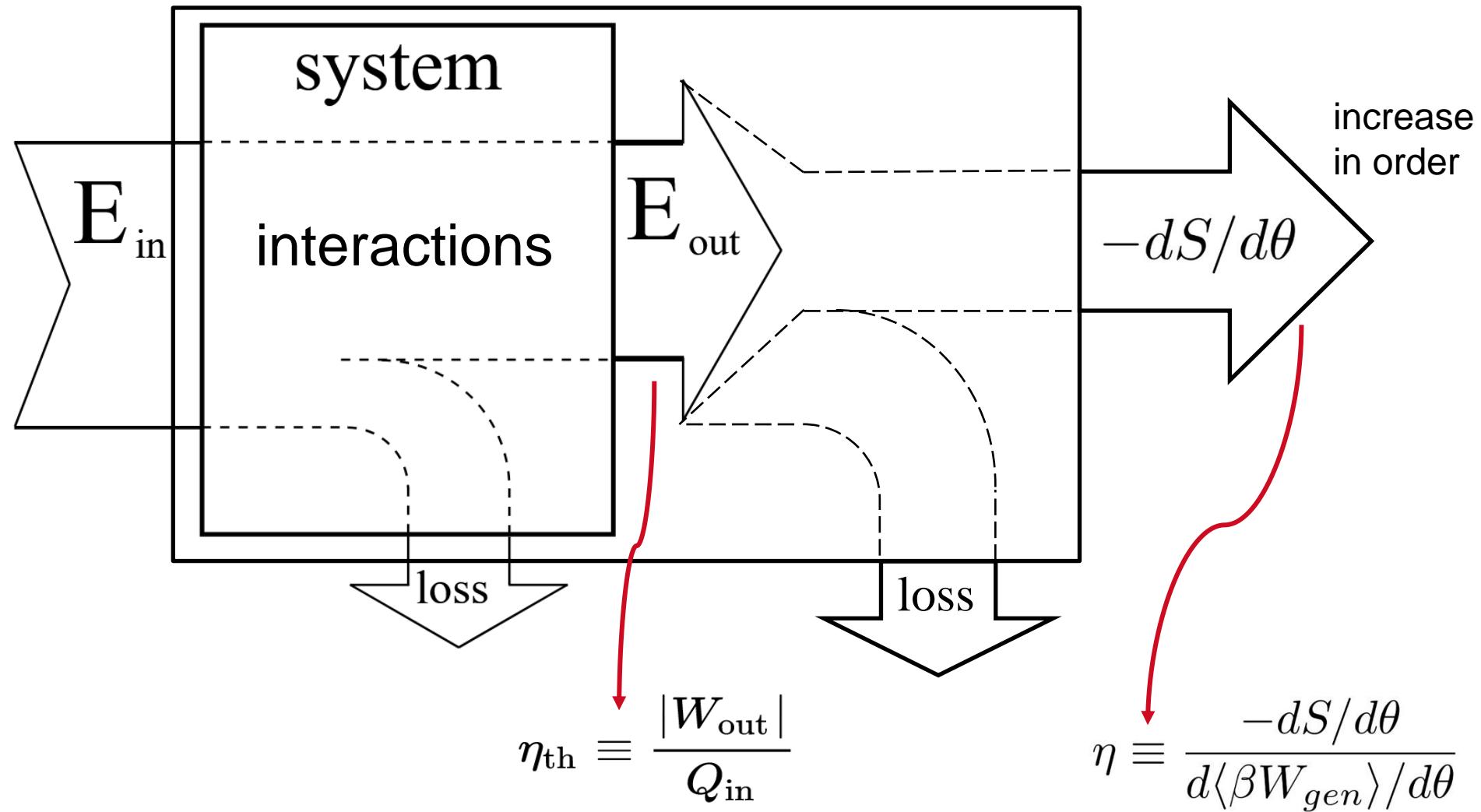
$$\eta_{th} \equiv \frac{\text{benefit}}{\text{cost}}$$

$$\eta_{th} \equiv \frac{|W_{out}|}{Q_{in}}$$



$$\eta_{\text{th}} \equiv \frac{|W_{\text{out}}|}{Q_{\text{in}}}$$

self-organisation



The reduction in uncertainty (the increase in order) from an expenditure of work for a given value of control parameter

$$\eta \equiv \frac{-dS/d\theta}{d\langle \beta W_{gen} \rangle / d\theta}$$



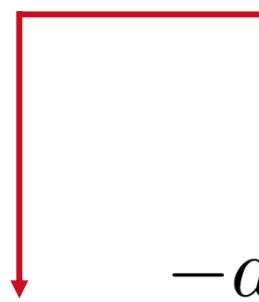
in thermodynamic
terms

Thermodynamic efficiency

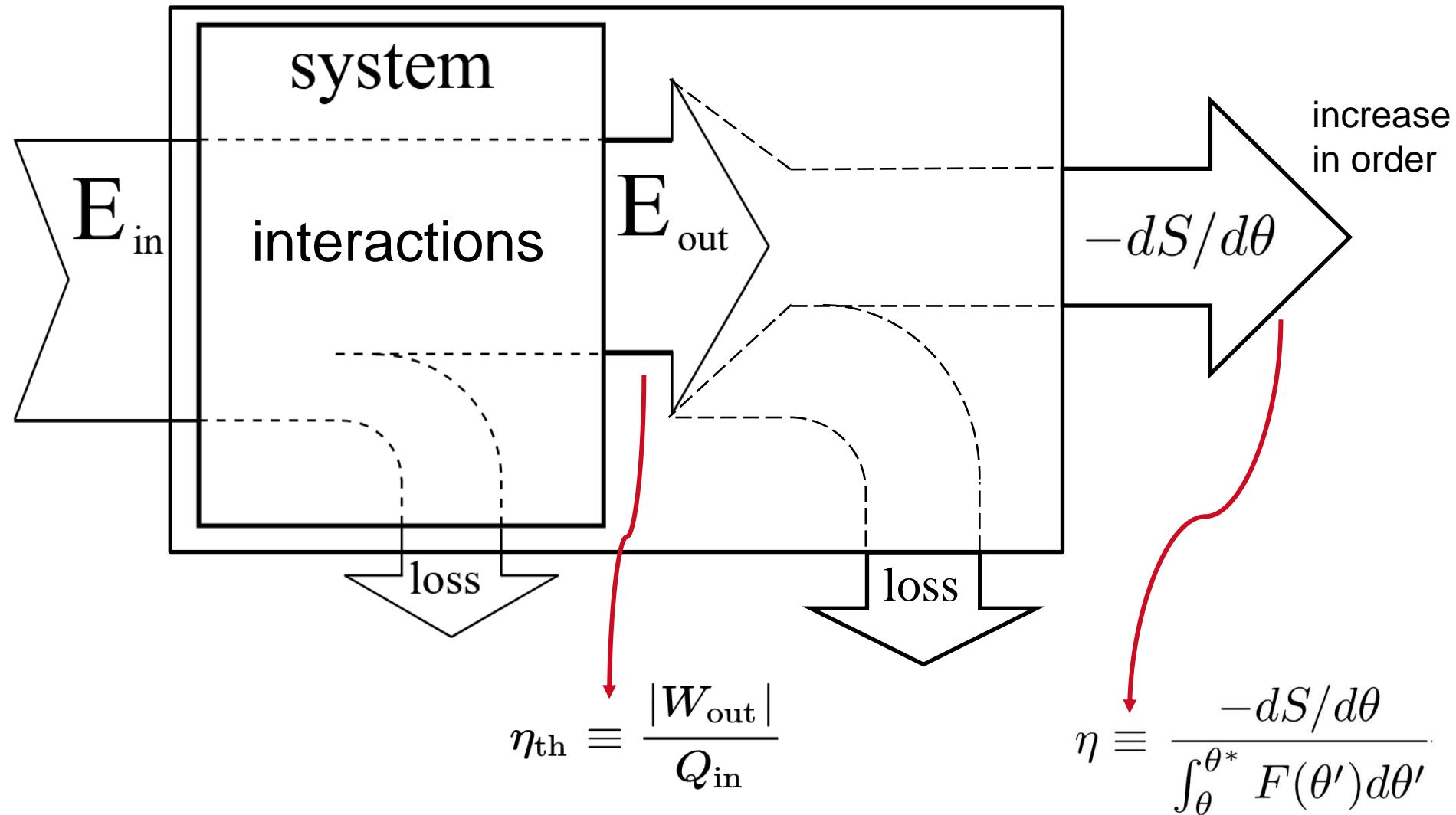
$$\eta \equiv \frac{-dS/d\theta}{d\langle \beta W_{gen} \rangle / d\theta} = \frac{-dS/d\theta}{\int_{\theta}^{\theta^*} F(\theta') d\theta'}$$

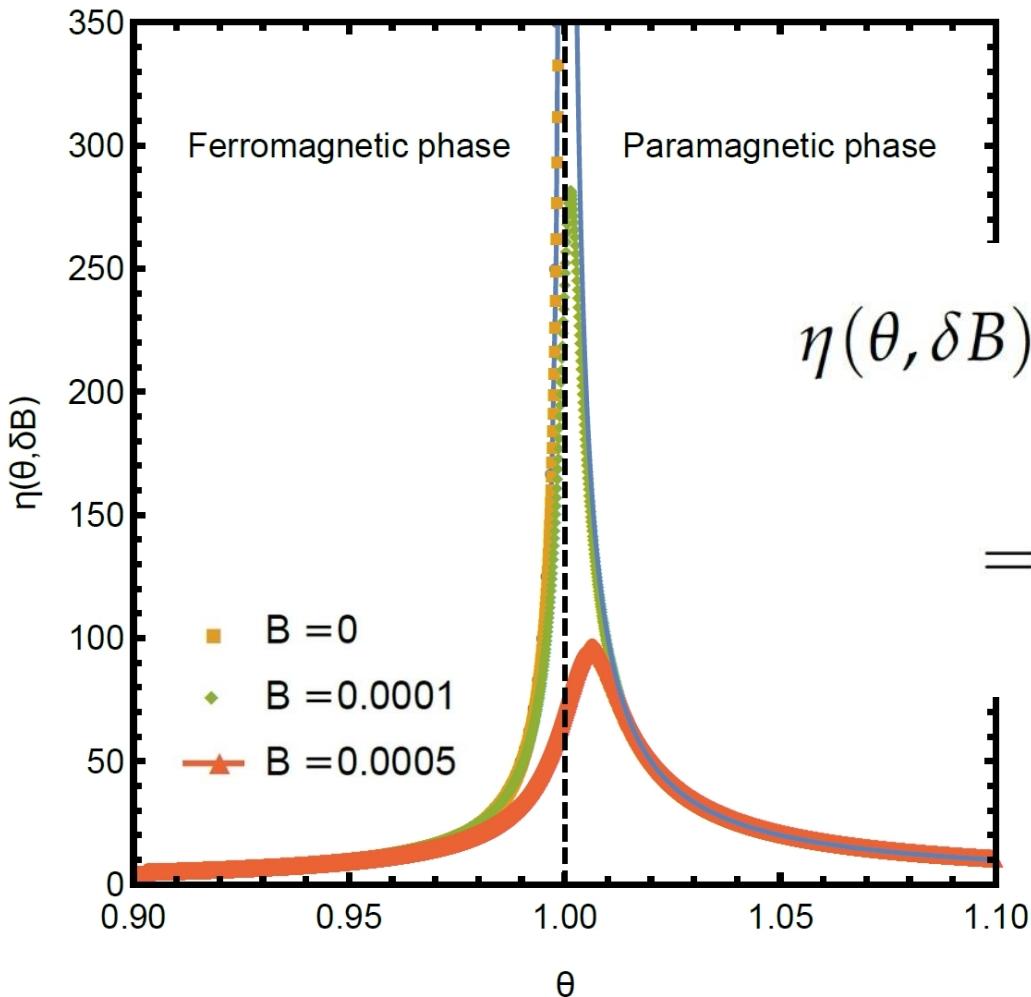
in thermodynamic
terms

in computational
terms



self-organisation





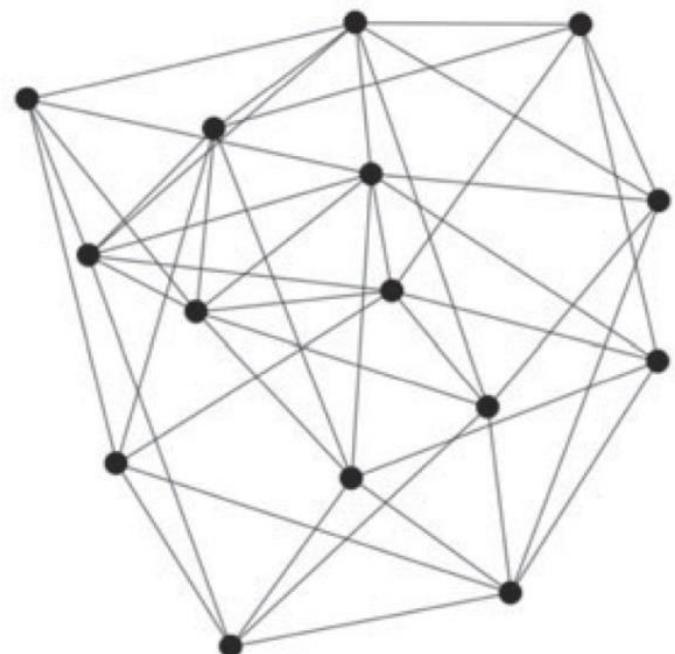
$$t \equiv (\theta - \theta_c)/\theta_c$$

$$\begin{aligned}\eta(\theta, \delta B) &= \frac{1}{k_B} \frac{\partial s}{\partial B} / \frac{\partial f}{\partial B} \\ &= \begin{cases} -\frac{1}{k_B} \frac{1}{2} t^{-1} & \text{for } t < 0, \\ \frac{1}{k_B \theta_c} t^{-1} & \text{for } t > 0. \end{cases}\end{aligned}$$

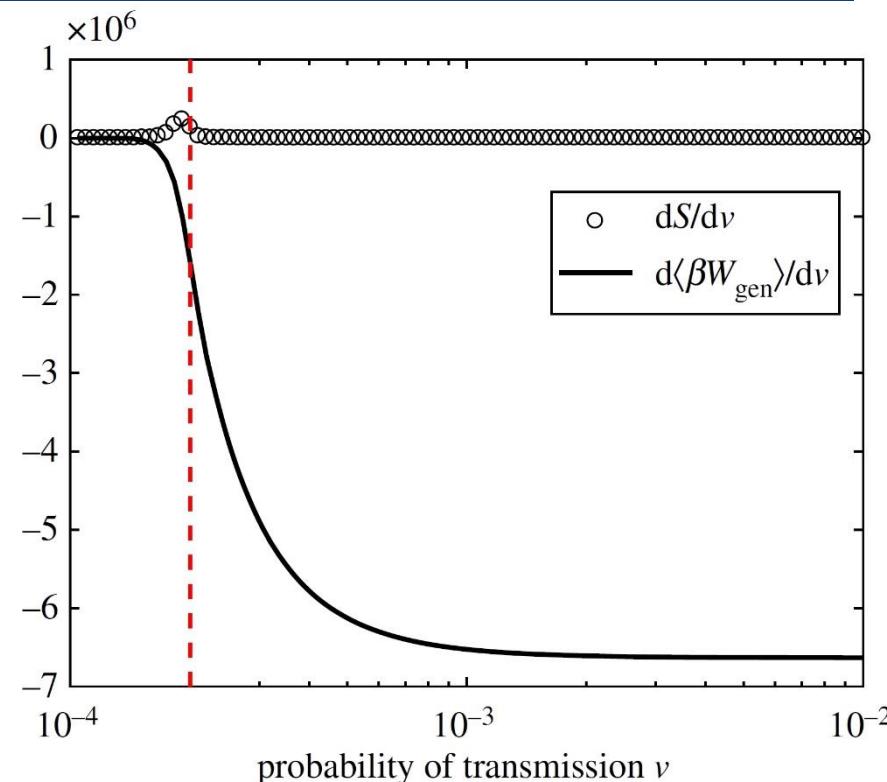
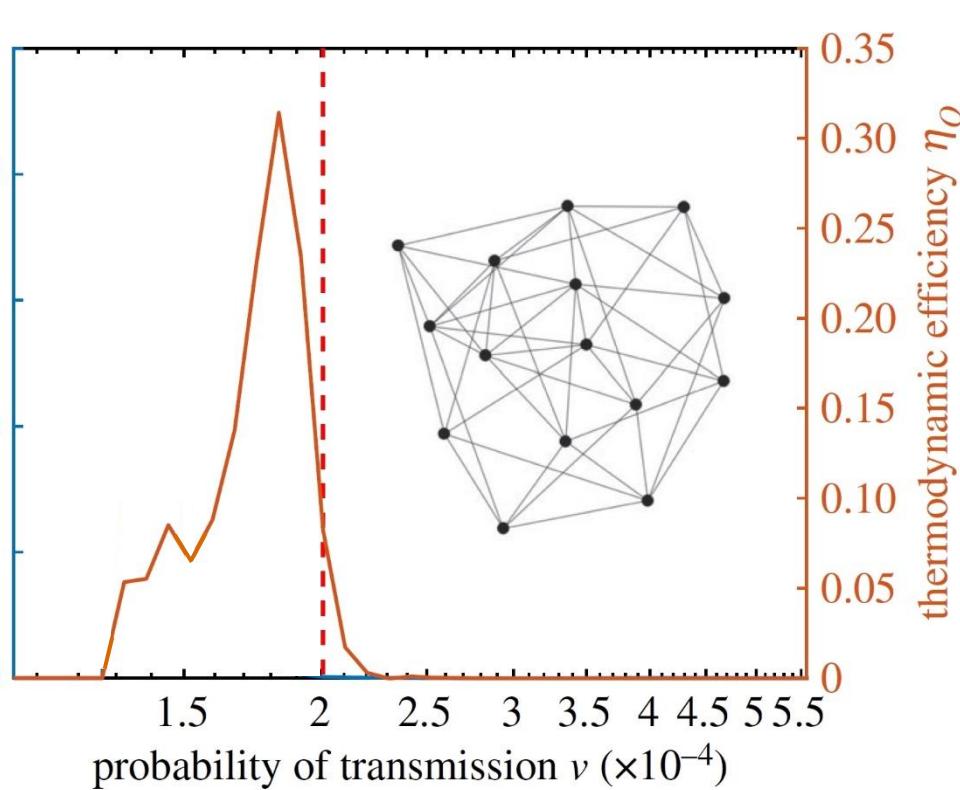


$$\left. \begin{aligned} \frac{dS}{dt} &= \gamma I - \beta IS \\ \frac{dI}{dt} &= \beta IS - \gamma I, \end{aligned} \right\} \quad \begin{aligned} \beta / \gamma &= R_0 \\ \nu & \\ \delta & \end{aligned}$$

$$R_0 = \frac{k\nu}{\nu + \delta - \nu\delta}$$



Epidemics as thermodynamic phenomena



- **intervention:** reducing the transmission probability, expending the work
- **pathogen emergence:** increasing the transmission probability, extracting the work

- Thermodynamic and computational perspectives:
 - rate of work carried out to change control parameter = accumulated sensitivity of distributed computation (integral of Fisher information)
- Thermodynamic efficiency:
 - the reduction in uncertainty (the increase in order) from an expenditure of work for a given value of control parameter
 - diverges at critical point for model systems (e.g., Ising model)

$$\eta(X, \delta X) = -\frac{1}{k_B} \frac{\beta}{|T - T_c|}$$

- *Principle of Super-efficiency:*
 - efficiency of self-organisation is maximal at critical points in dynamical systems

- O. M. Cliff, N. Harding, M. Piraveenan, E. Y. Erten, M. Gambhir, M. Prokopenko, Investigating Spatiotemporal Dynamics and Synchrony of Influenza Epidemics in Australia: An Agent-Based Modelling Approach, *Simulation Modelling Practice and Theory*, 87, 412-431, 2018.
- K. M. Fair, C. Zachreson, M. Prokopenko, Creating a surrogate commuter network from Australian Bureau of Statistics census data, *Scientific Data*, 6: 150, 2019.
- C. Zachreson, K. M. Fair, N. Harding, M. Prokopenko, Interfering with influenza: nonlinear coupling of reactive and static mitigation strategies, *Journal of Royal Society Interface*, 17(165): 20190728, 2020.
- N. Harding, R. E. Spinney, M. Prokopenko, Population mobility induced phase separation in SIS epidemic and social dynamics, *Scientific Reports*, 10: 7646, 2020.
- N. Harding, R. Nigmatullin, M. Prokopenko, Thermodynamic efficiency of contagions: a statistical mechanical analysis of the SIS epidemic model, *Interface Focus*, 8 20180036, 2018.
- M. Prokopenko, J. T. Lizier, O. Obst, X. R. Wang, Relating Fisher information to order parameters, *Physical Review E*, 84, 041116, 2011.
- E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, *Physical Review E*, 97, 012120, 2018.
- R. Nigmatullin, M. Prokopenko, Thermodynamic efficiency of interactions in self-organizing systems, *Entropy*, 23(6): 757, 2021.