

Survival of the “super-efficient”: Thermodynamic efficiency of self-organisation

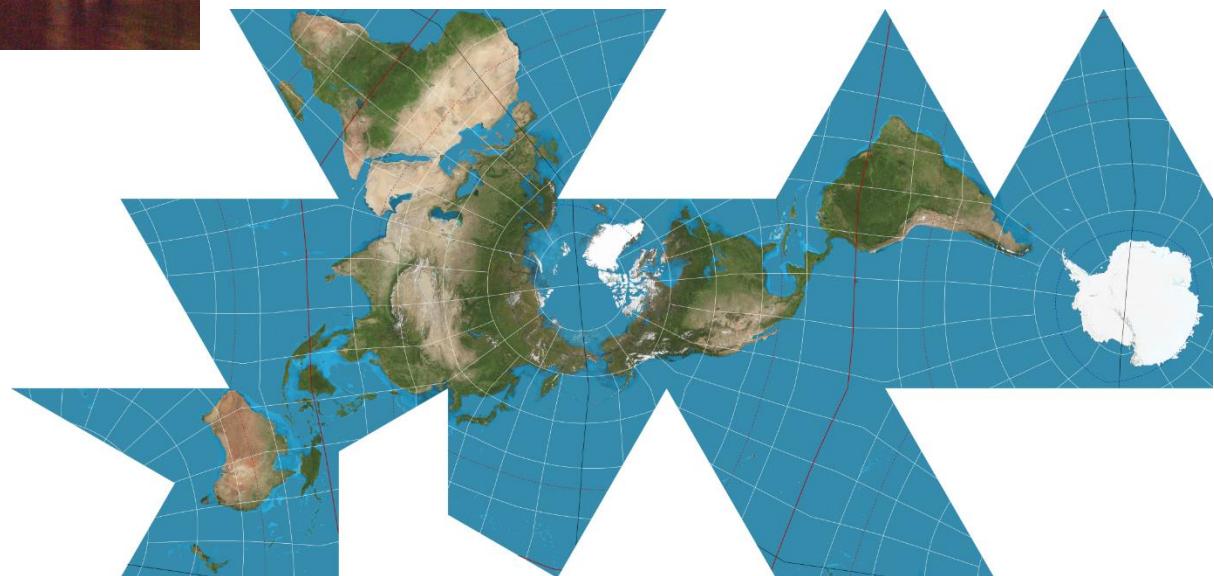
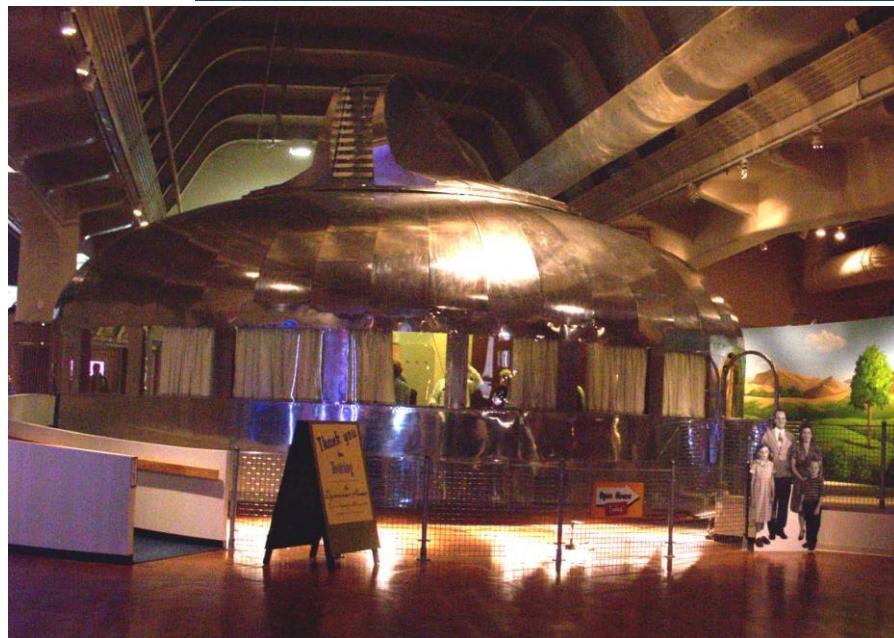
Prof. Mikhail Prokopenko
Centre for Complex Systems
School of Computer Science, Faculty of Engineering
Sydney Institute for Infectious Diseases



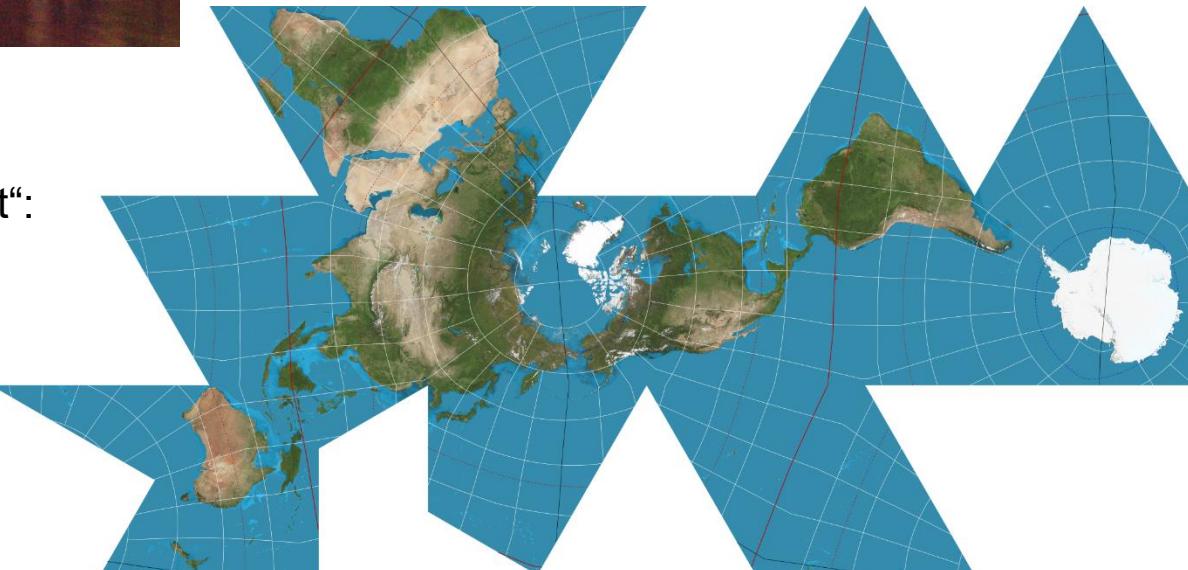
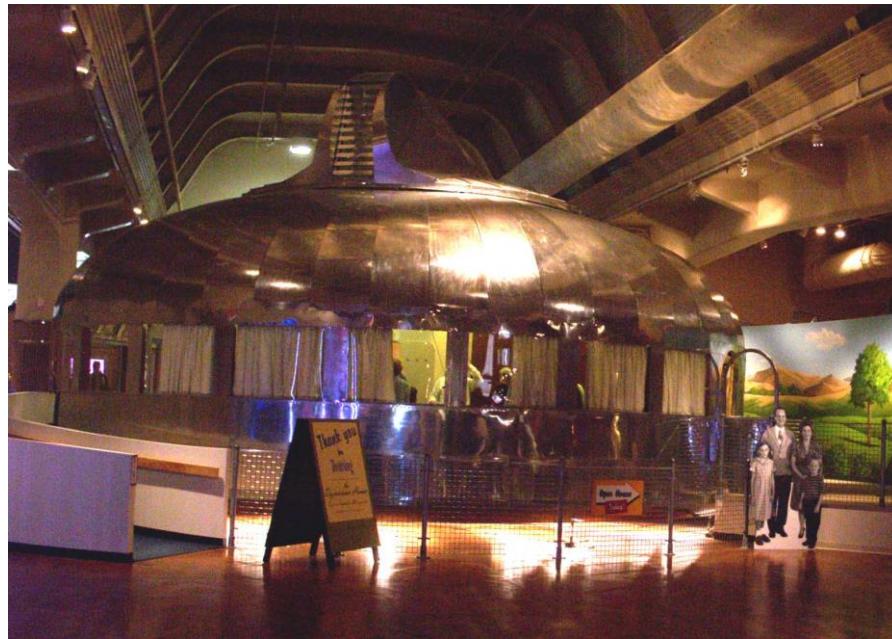


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Teaser...



Dymaxion: house, car, map projection



Dymaxion: "maximum gain of advantage from minimal energy input":

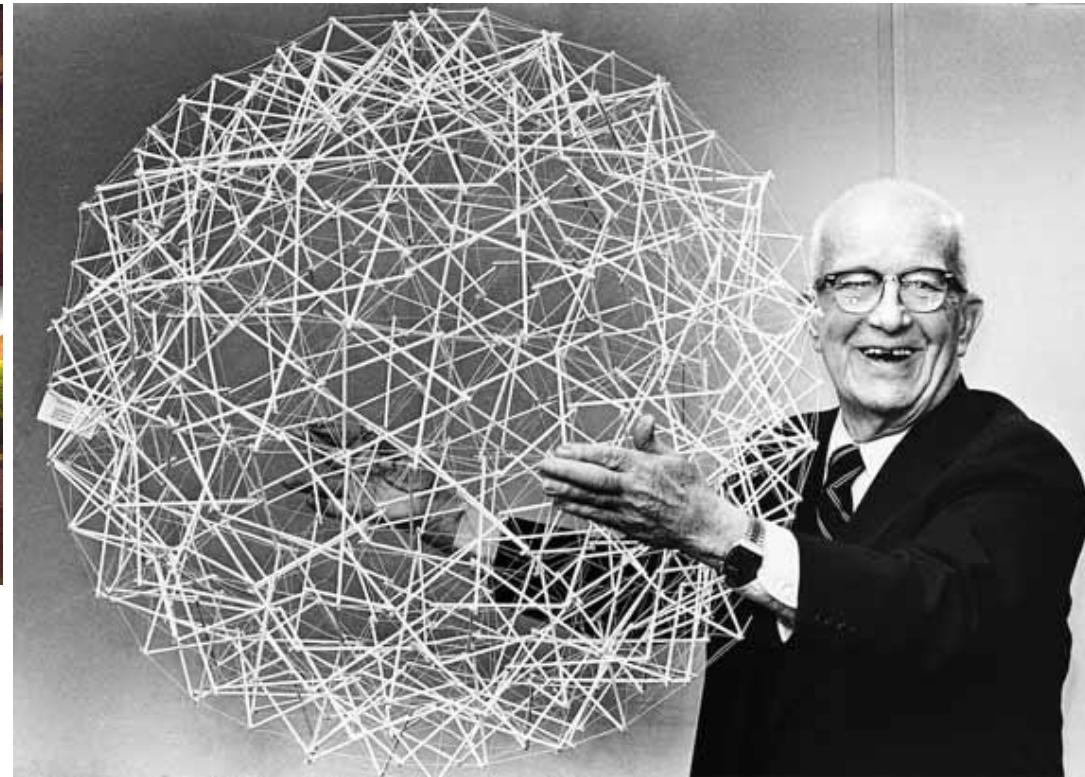
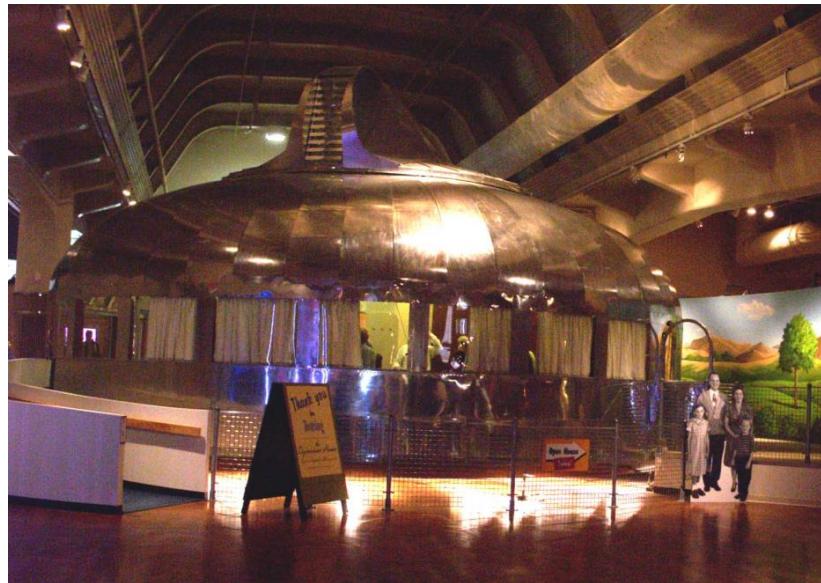
DYnamic, **MAX**imum and tens**ION**

(Buckminster Fuller)



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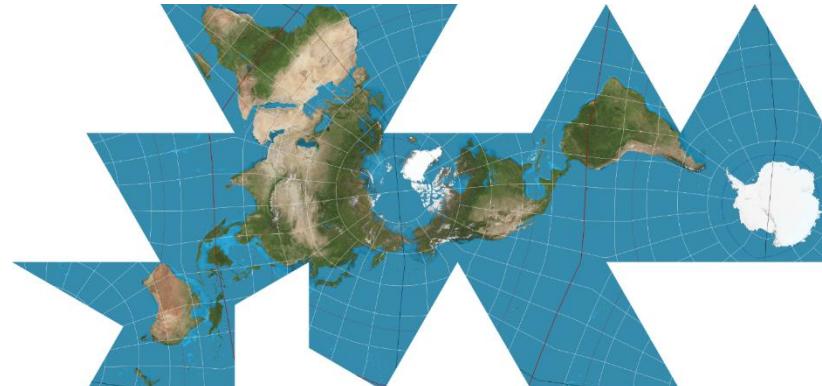
Dymaxion: Buckminster Fuller



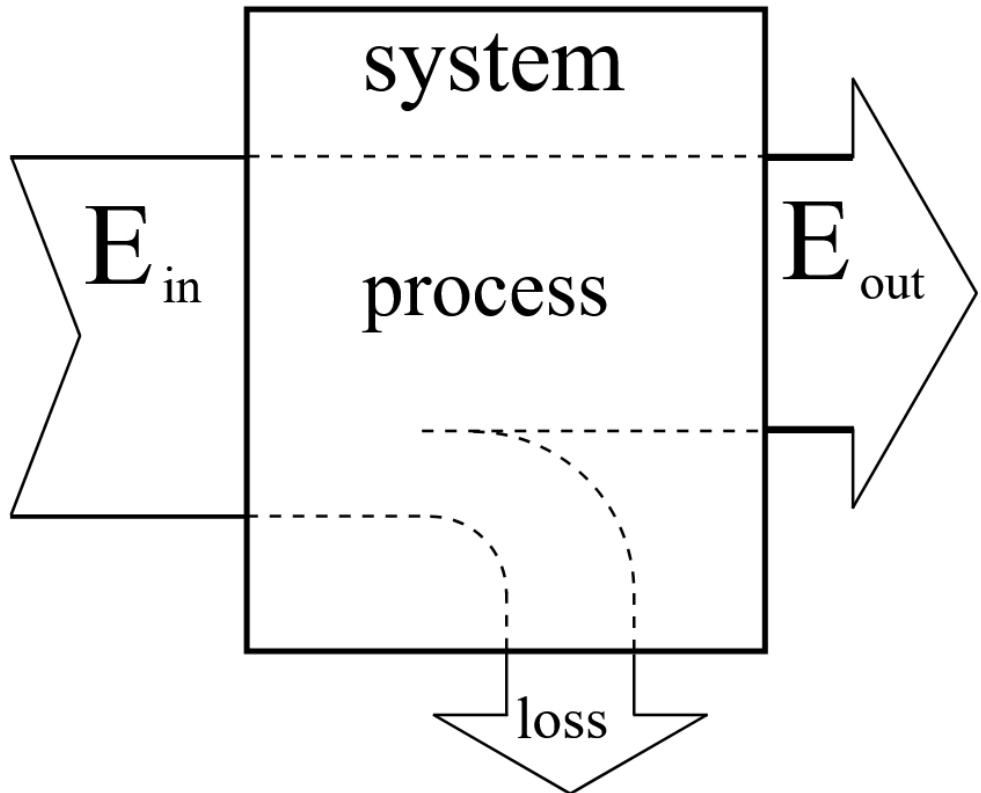
Dymaxion: "maximum gain of advantage from minimal energy input":

DYnamic, **MAX**imum and tens**ION**

(Buckminster Fuller)



- Thermal vs thermodynamic efficiency
- Criticality and phase transitions
- Fisher information: information theory and thermodynamics
- Case studies:
 - collective / swarming motion (*Physical Review E*, 2018)
 - urban dynamics (*Royal Society Open Science*, 2018)
 - epidemic dynamics (*Royal Society Interface Focus*, 2018)
 - Curie-Weiss Ising model (*Entropy*, 2021)



$$Q_{in} = |W_{out}| + |Q_{out}|$$

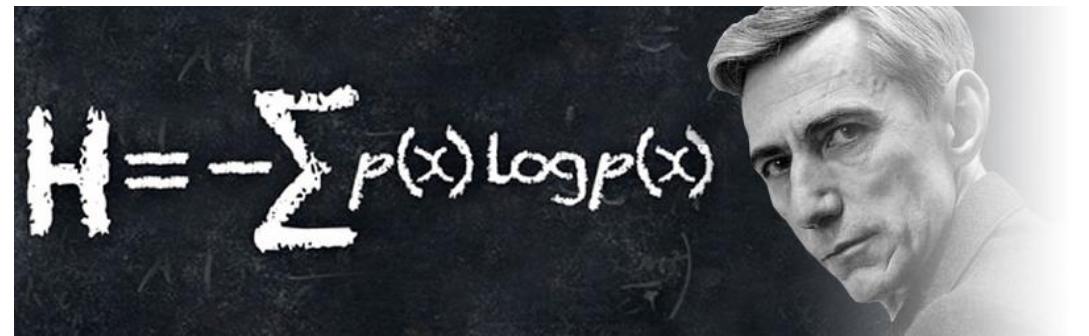
$$\eta_{th} \equiv \frac{\text{benefit}}{\text{cost}}$$

$$\eta_{th} \equiv \frac{|W_{out}|}{Q_{in}}$$

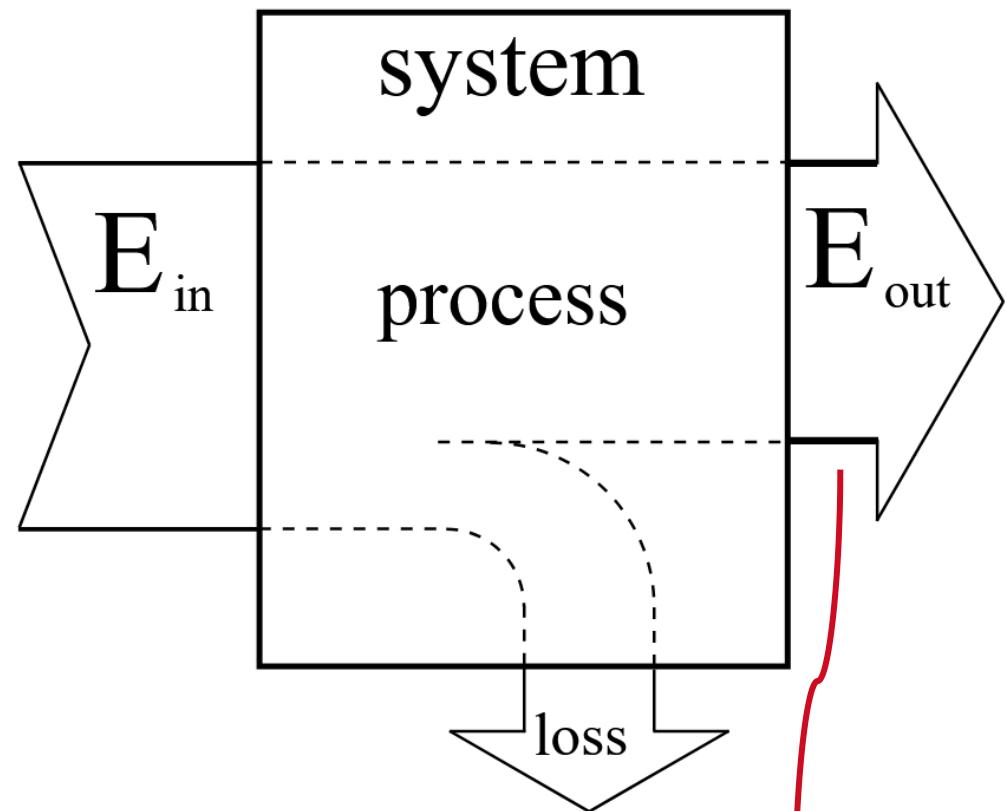


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Information and entropy

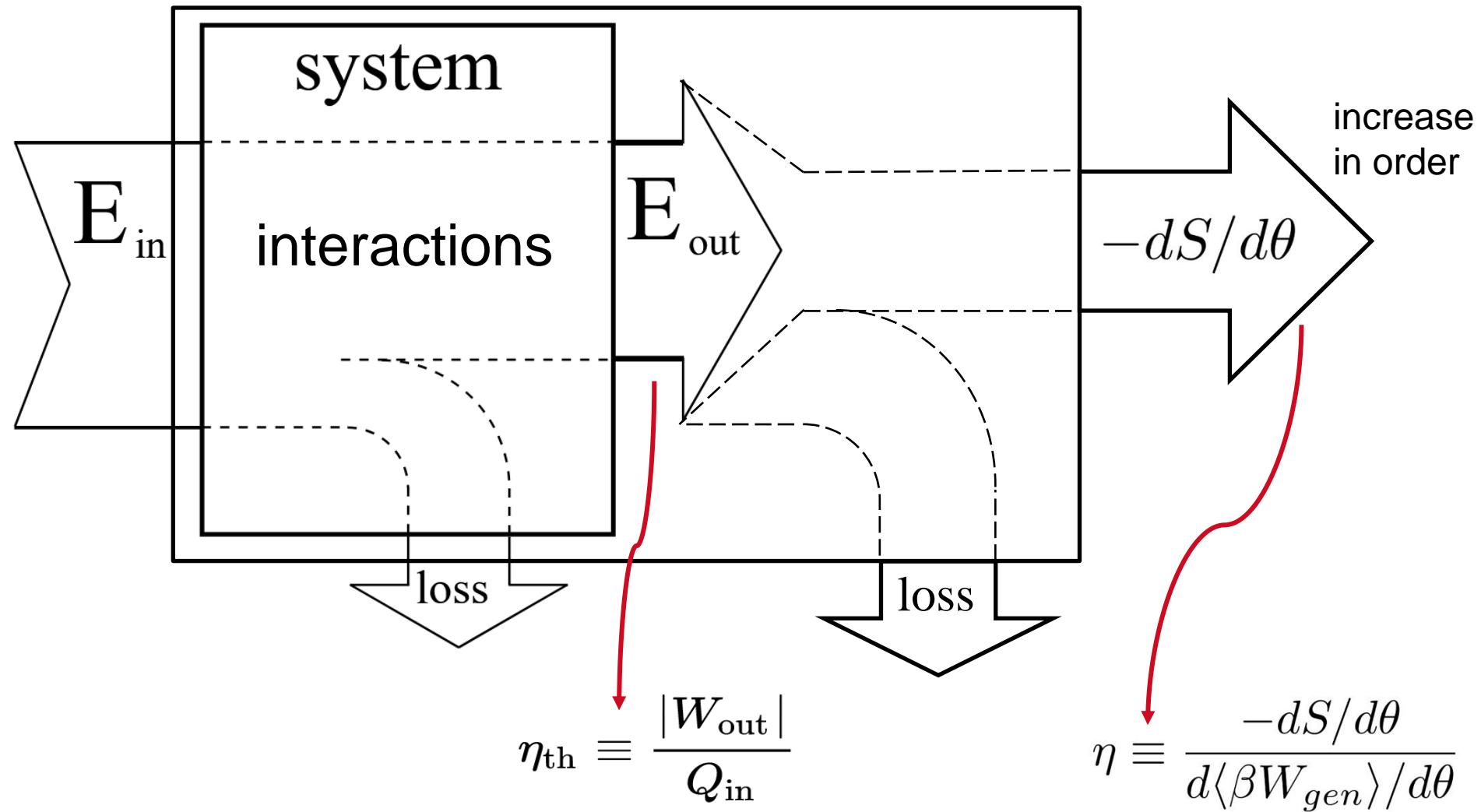


entropy



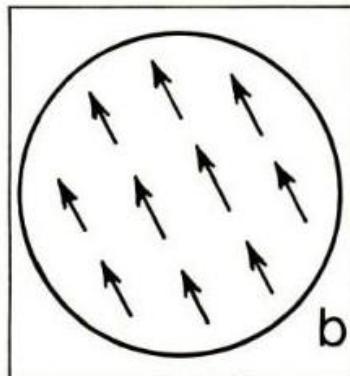
$$\eta_{th} \equiv \frac{|W_{out}|}{Q_{in}}$$

self-organisation

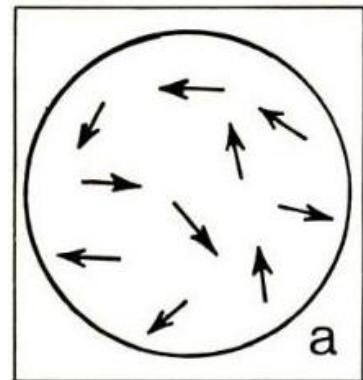




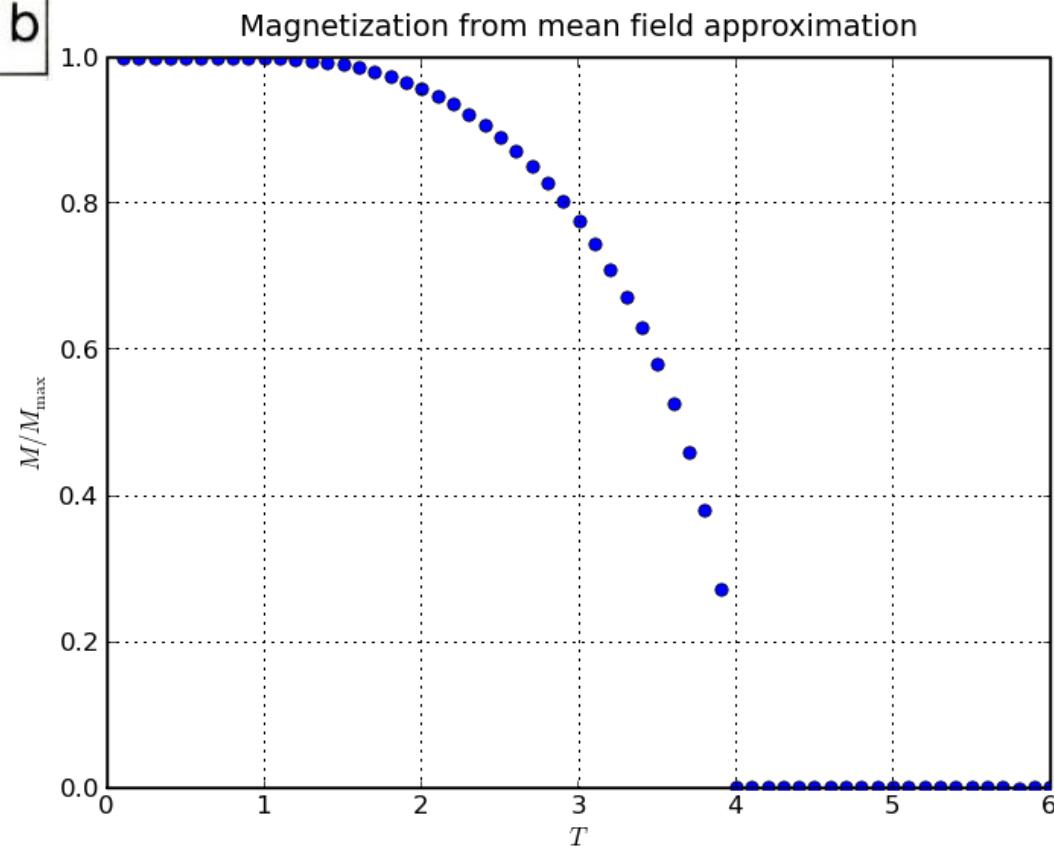
Phase transitions and order parameters



anisotropic
“coherent”

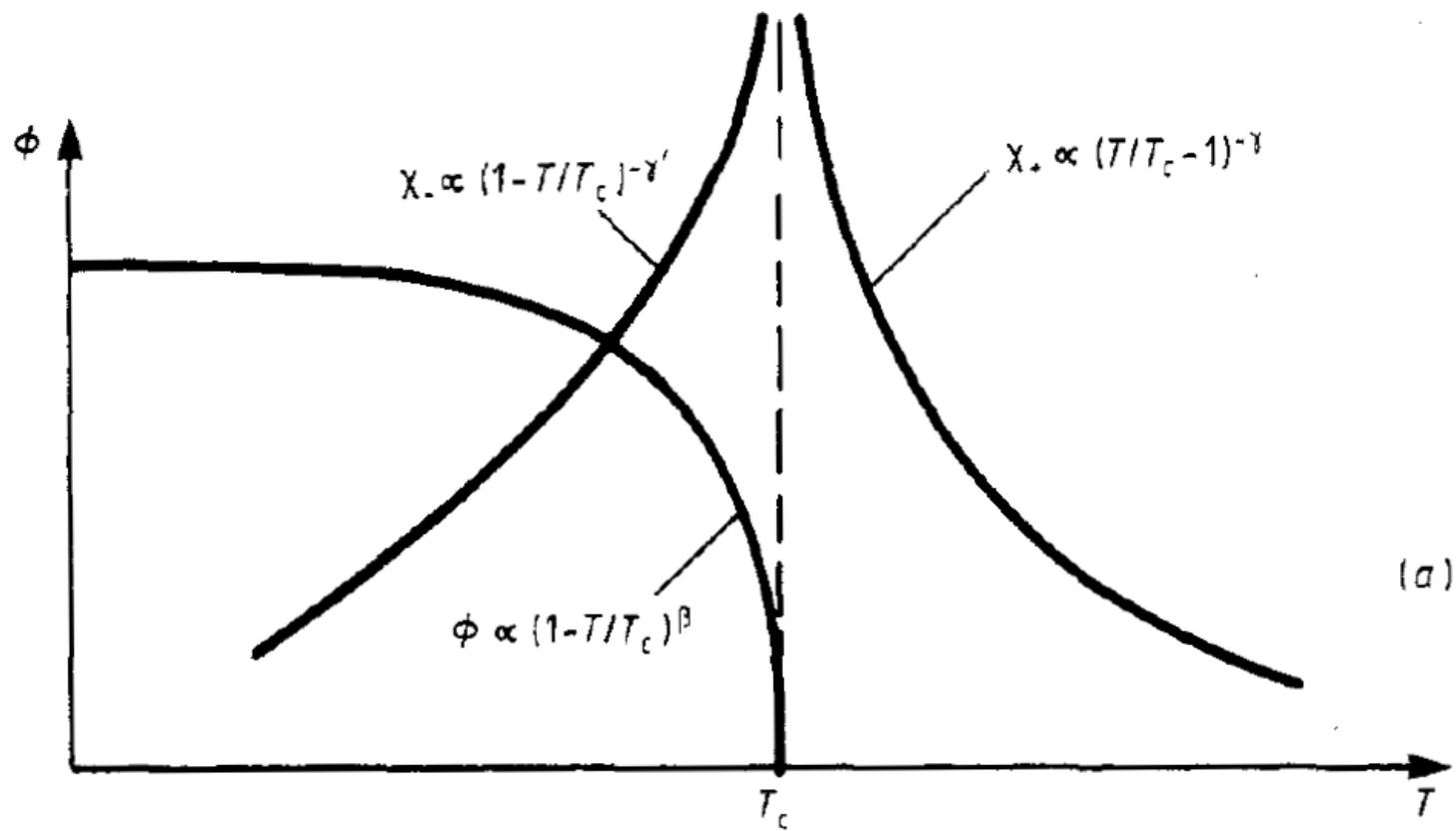


isotropic
“disordered”



Derivative of order parameter (divergence)

K Binder (1987)

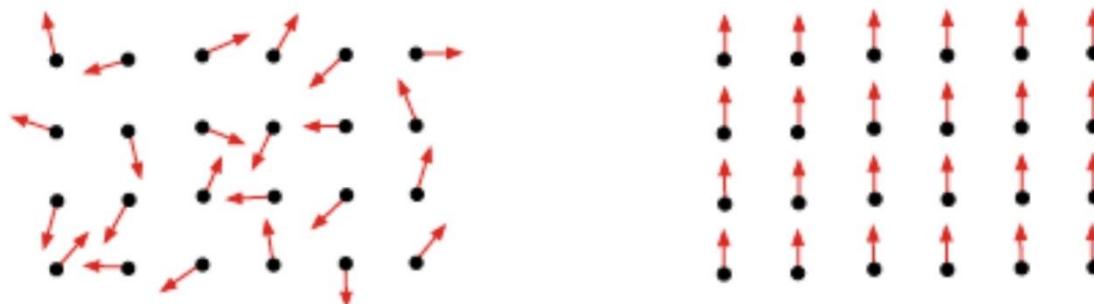


Fisher Information and sensitivity

A way of measuring the amount of information that an observable random variable X has about an unknown parameter θ



$$F(\theta) = \int_x \left(\frac{\partial \ln(p(x|\theta))}{\partial \theta} \right)^2 p(x|\theta) dx$$

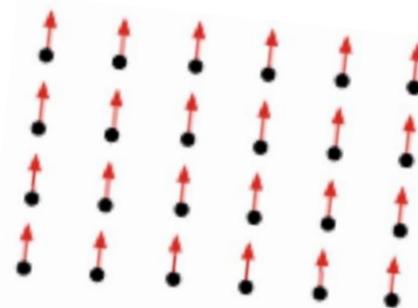
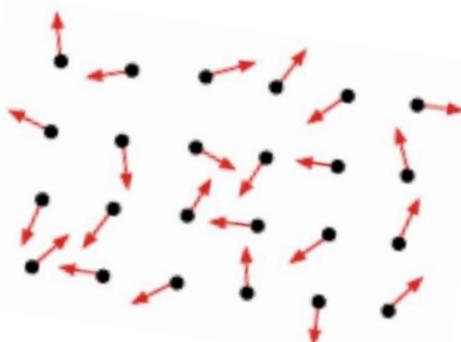


Fisher Information and sensitivity

A way of measuring the amount of information that an observable random variable X has about an unknown parameter θ



$$F(\theta) = \int_x \left(\frac{\partial \ln(p(x|\theta))}{\partial \theta} \right)^2 p(x|\theta) dx$$



$$G(T, \theta_i) = U(S, \phi_i) - TS - \phi_i \theta_i$$

$$F_{ij}(\theta) = \beta \frac{\partial \phi_i}{\partial \theta_j}$$

Fisher information matrix

Rate of change of the
order parameter

Fisher Information and generalised work

- A way of measuring the amount of information that an observable random variable X has about an unknown parameter θ

$$F(\theta) = \int_x \left(\frac{\partial \ln(p(x|\theta))}{\partial \theta} \right)^2 p(x|\theta) dx$$

- Fisher information is proportional to the curvature of the work in quasi-static processes

$$F(\theta) = - \frac{d^2 \langle \beta W_{gen} \rangle}{d\theta^2}$$

The reduction in uncertainty (the increase in order) from an expenditure of work for a given value of control parameter

$$\eta \equiv \frac{-dS/d\theta}{d\langle \beta W_{gen} \rangle / d\theta}$$



in thermodynamic
terms

Thermodynamic efficiency

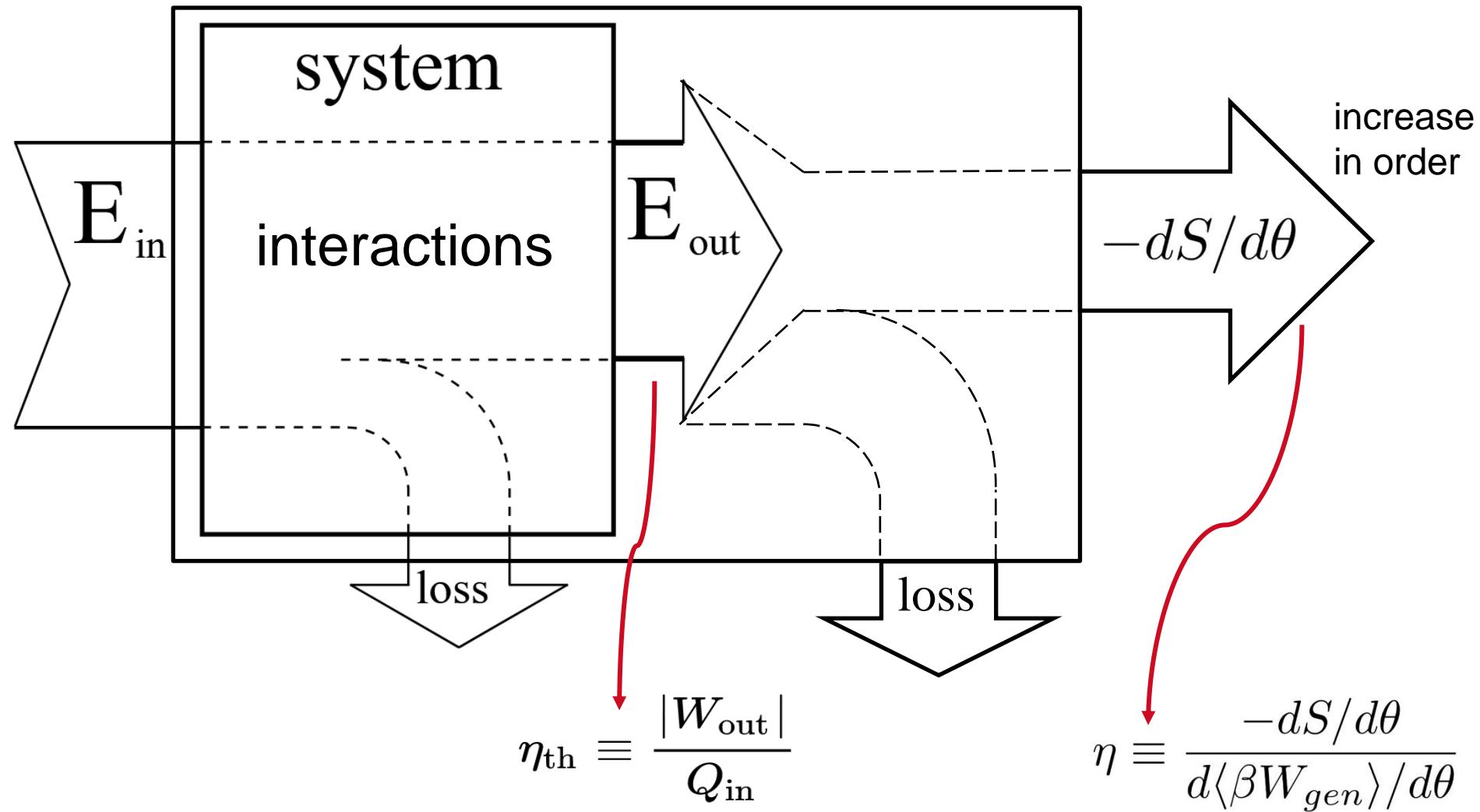
$$\eta \equiv \frac{-dS/d\theta}{d\langle \beta W_{gen} \rangle / d\theta} = \frac{-dS/d\theta}{\int_{\theta}^{\theta^*} F(\theta') d\theta'}$$

in thermodynamic
terms

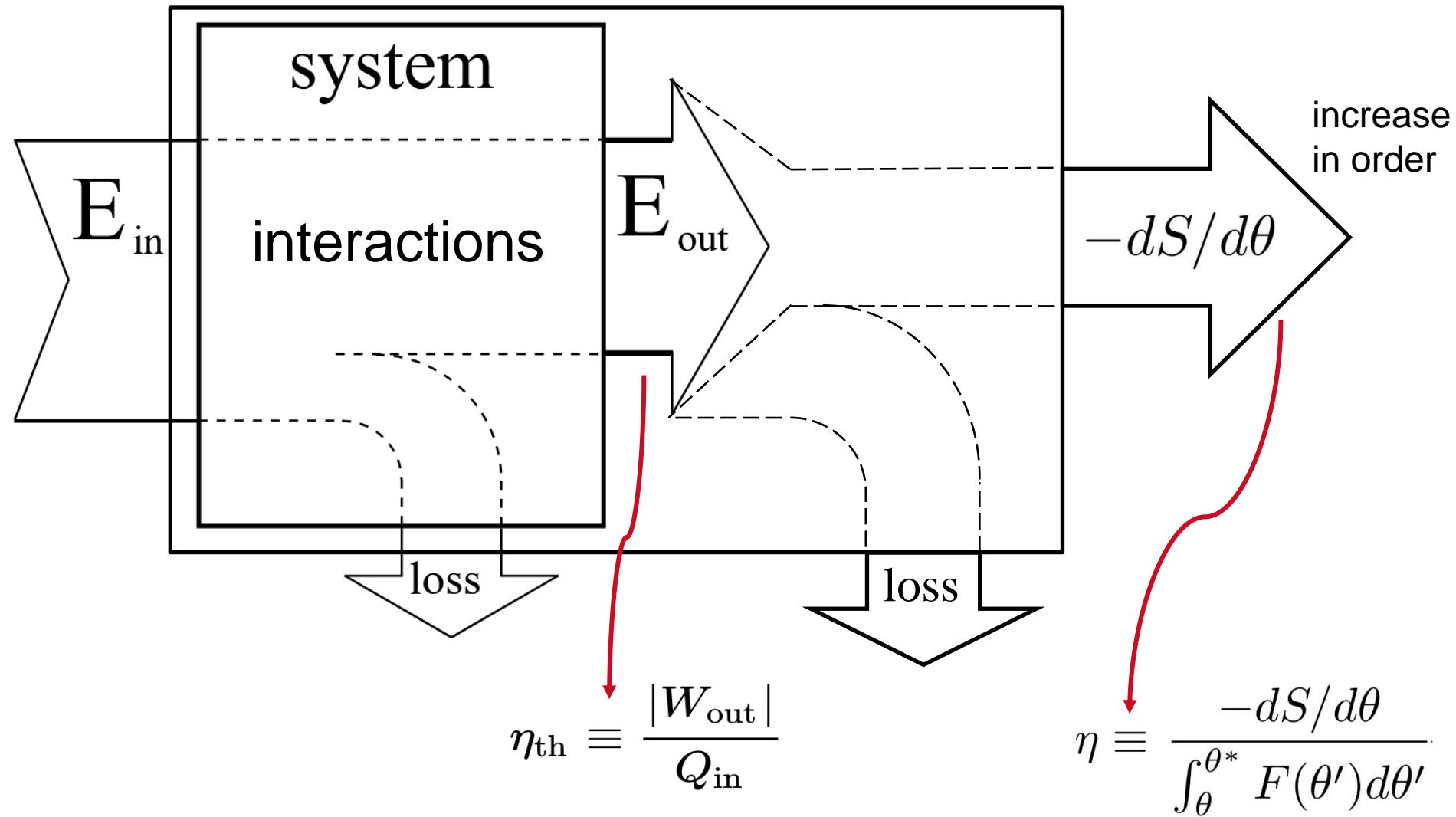
in computational
terms



self-organisation



self-organisation



1. A dynamical model of collective motion

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PHYSICAL REVIEW LETTERS

week ending
16 JANUARY 2004

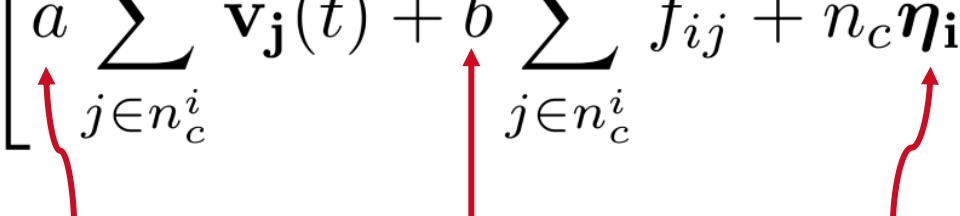
Onset of Collective and Cohesive Motion

Guillaume Grégoire and Hugues Chaté

*CEA—Service de Physique de l’État Condensé, CEN Saclay, 91191 Gif-sur-Yvette, France
 Pôle Matière et Systèmes Complexes, CNRS FRE 2348, Université de Paris VII, Paris, France*
 (Received 12 August 2003; published 15 January 2004)

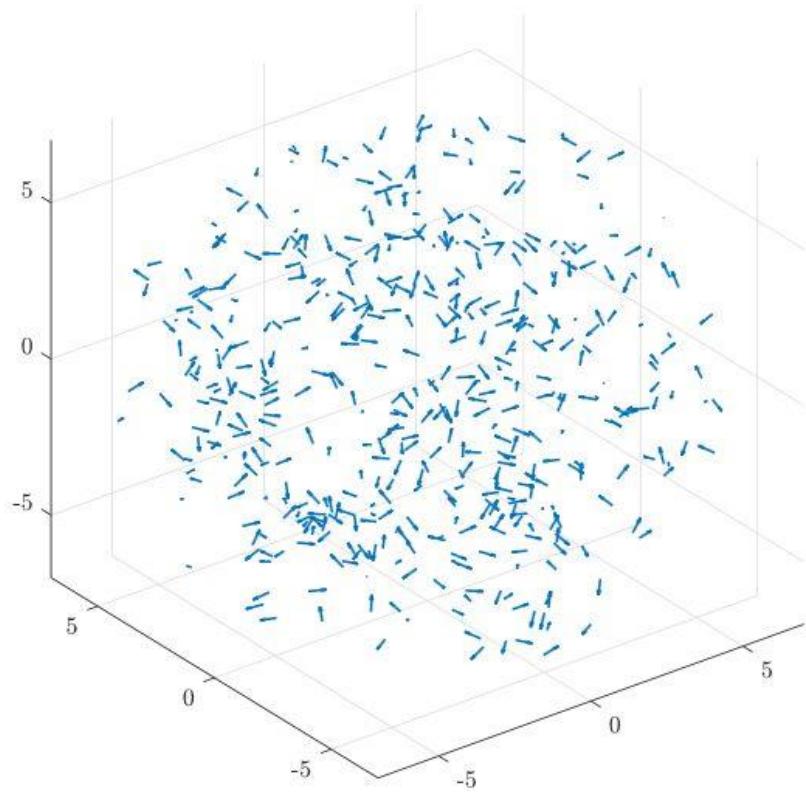
$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t)$$

$$\mathbf{v}_i(t+1) = v_0 \Theta \left[a \sum_{j \in n_c^i} \mathbf{v}_j(t) + b \sum_{j \in n_c^i} f_{ij} + n_c \boldsymbol{\eta}_i \right]$$

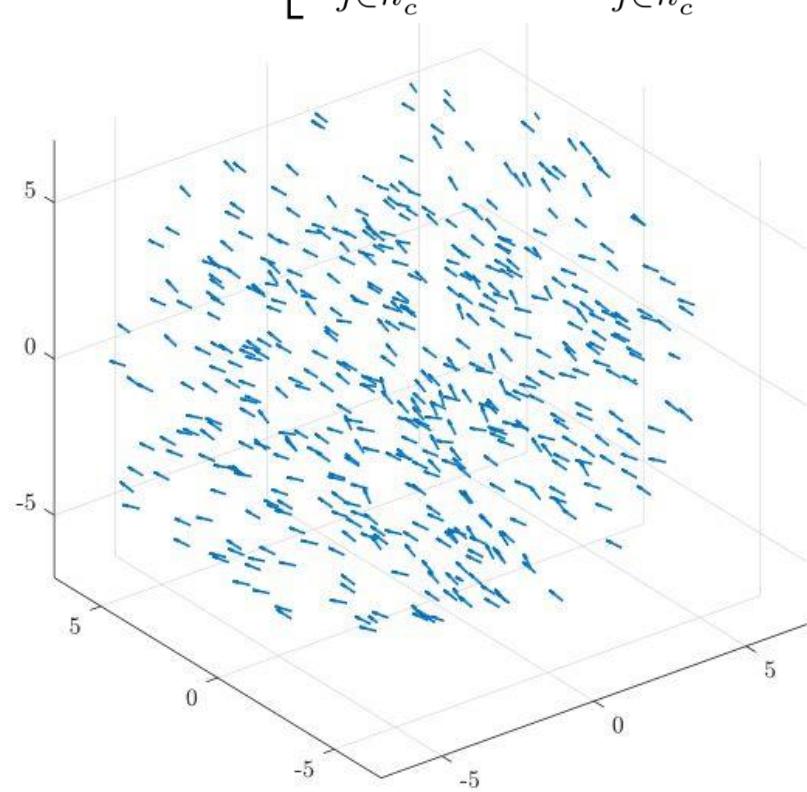

 alignment cohesion perturbation

Swarming (collective) motion

$$\mathbf{v}_i(t+1) = v_0 \Theta \left[a \sum_{j \in n_c^i} \mathbf{v}_j(t) + b \sum_{j \in n_c^i} f_{ij} + n_c \boldsymbol{\eta}_i \right]$$



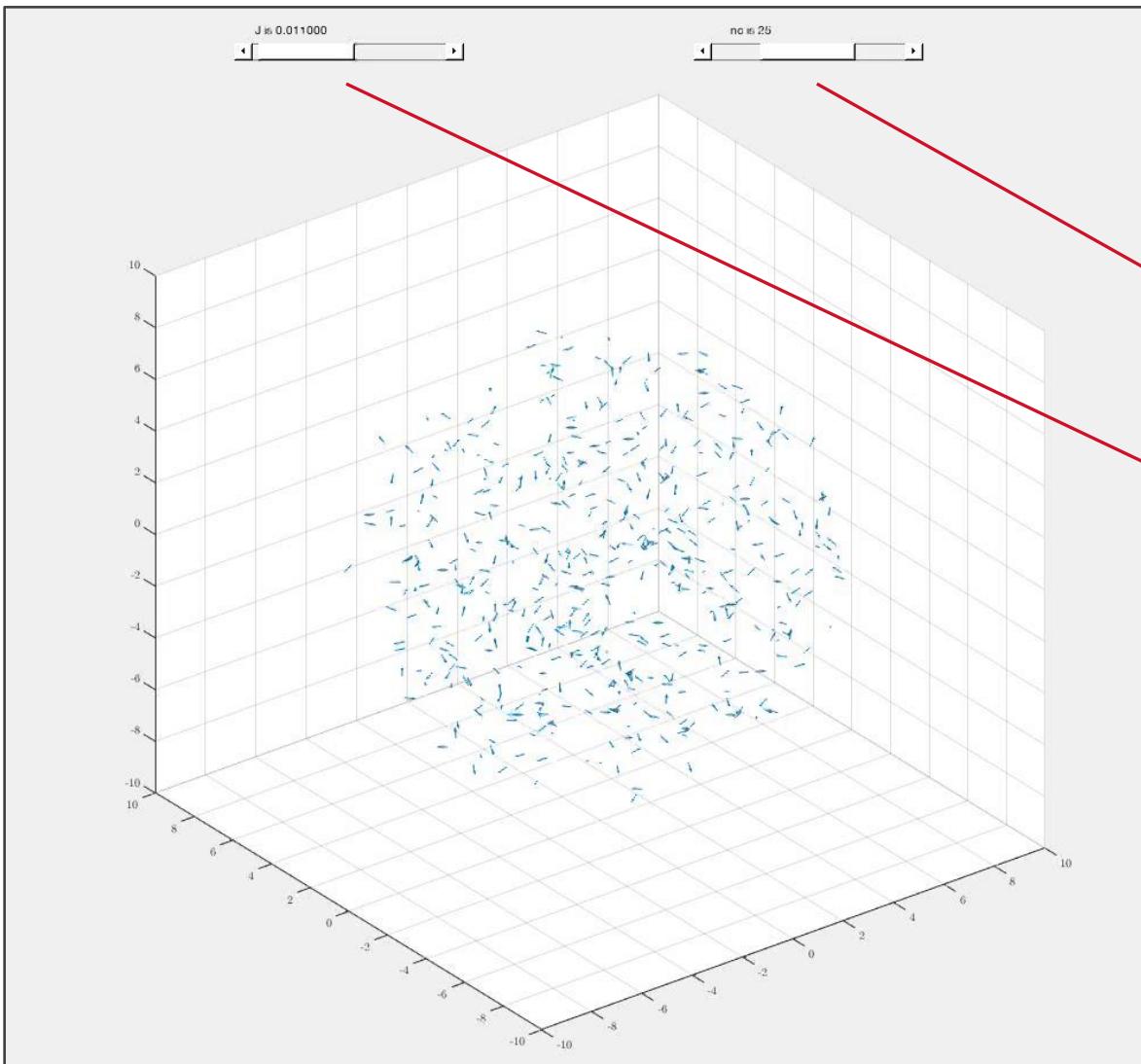
(a) $J = 0.001, n_c = 20$



(b) $J = 0.2, n_c = 20$



Varying control parameters

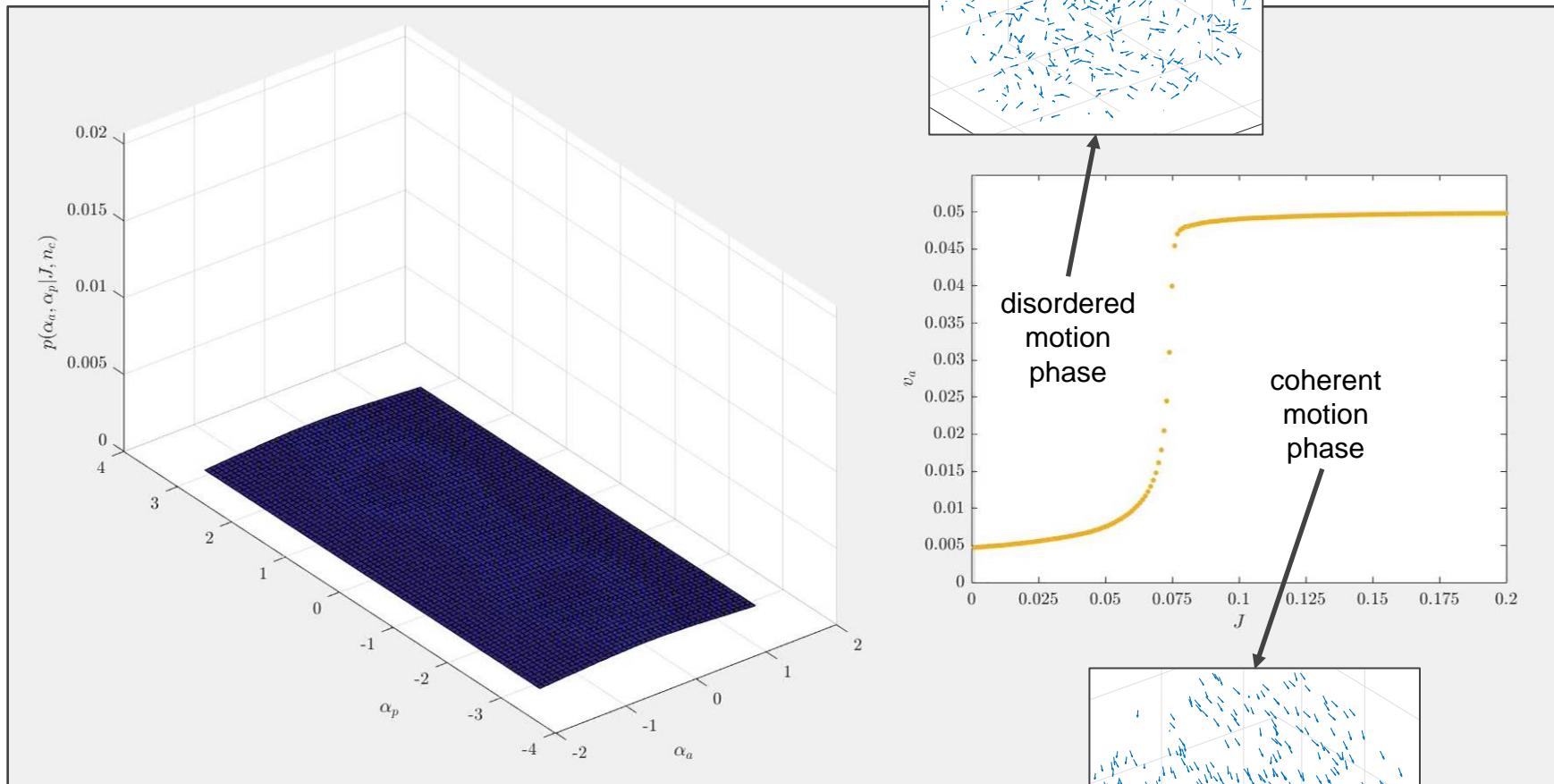


a kinetic phase
transition driven by

- nearest neighbours N_c
- alignment strength $J = v_0 a$

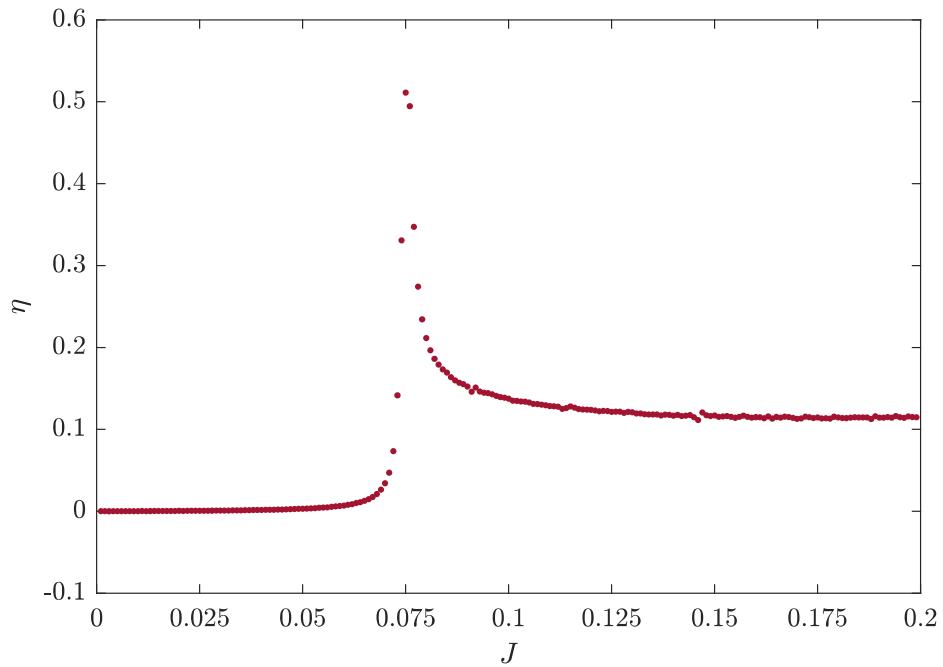
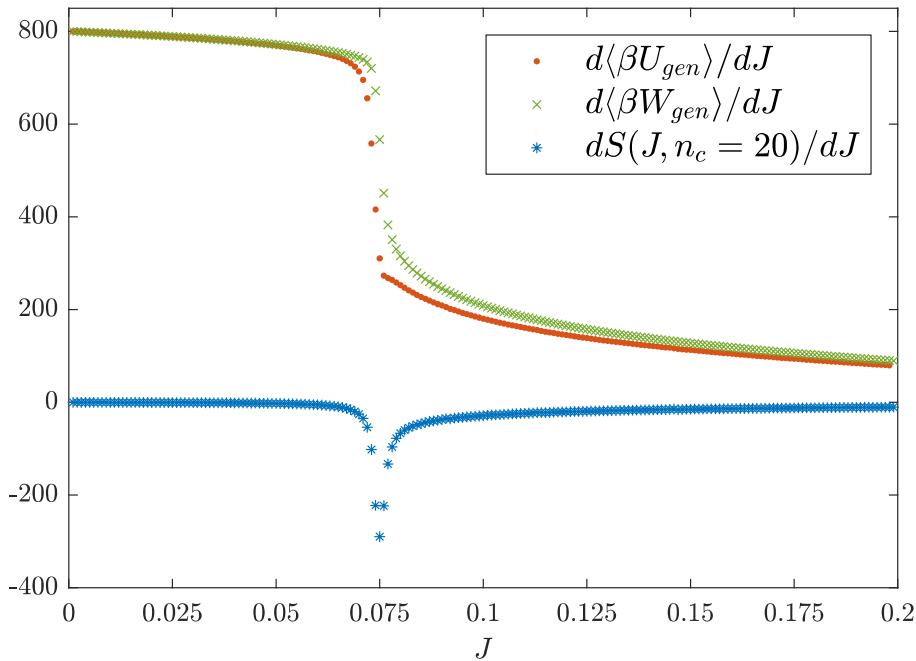


The two kinetic phases

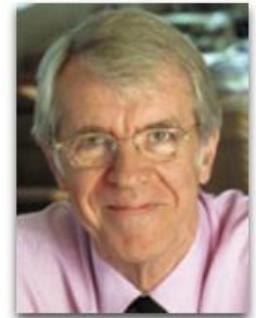
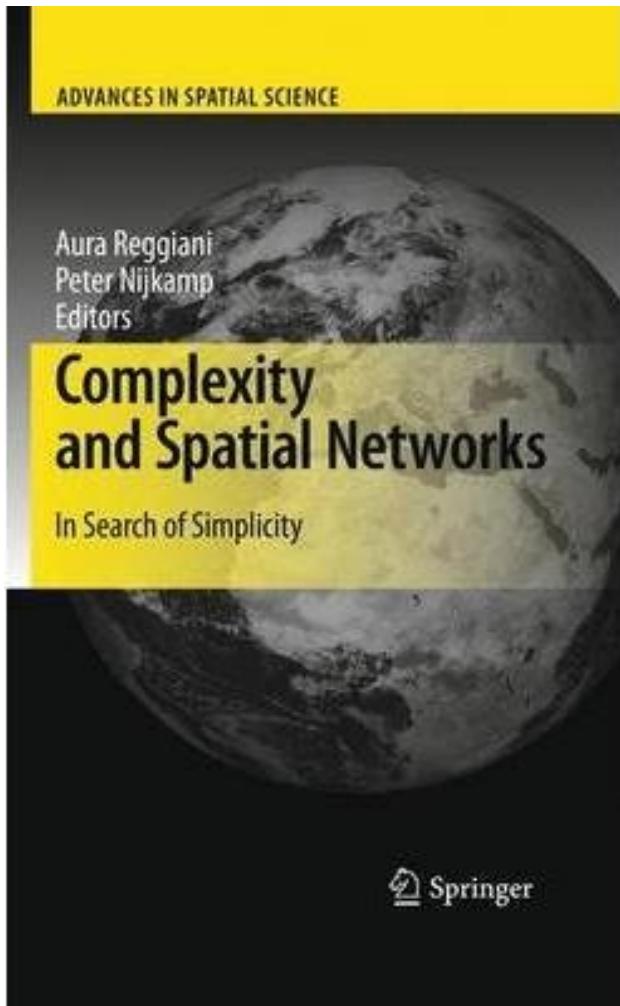


Thermodynamic efficiency of swarming behaviour

$$\eta = \frac{-dS(J, n_c)/dJ}{d\langle \beta W_{gen} \rangle/dJ}$$



2. “Thermodynamics” and “evolution” of the city



Chapter 2 The “Thermodynamics” of the City| Evolution and Complexity Science in Urban Modelling

Alan Wilson

2009

Maximum Entropy (MaxEnt) Principle

Reprinted from THE PHYSICAL REVIEW, Vol. 106, No. 4, 620-630, May 15, 1957
Printed in U. S. A.



Information Theory and Statistical Mechanics

E. T. JAYNES

Department of Physics, Stanford University, Stanford, California

(Received September 4, 1956; revised manuscript received March 4, 1957)

What is Maximum Entropy (MaxEnt) Principle

1. **MaxEnt** is a technique for automatically acquiring probabilistic knowledge from incomplete information without making any unsubstantiated assumptions. **Entropy** is a mathematical measure of uncertainty or ignorance: greater **entropy** corresponds to greater ignorance. Hence, the **MaxEnt** solution is the least biased possible solution given whatever is experimentally known, but assuming nothing else.

Applying MaxEnt Principle: urban wealth

What is the least biased distribution \mathcal{M}_{ij} (to be compared with the data distribution M_{ij}) that maximises the “entropy”

$$H = - \sum_i \sum_j \mathcal{M}_{ij} \log \mathcal{M}_{ij} \quad (1)$$

subject to the following constraints:

$$\sum_j \mathcal{M}_{ij} = M_i^e, \quad (2)$$

$$\sum_i \sum_j \mathcal{M}_{ij} \cdot A_j = A^{tot}, \quad (3)$$

$$\sum_i \sum_j \mathcal{M}_{ij} \cdot c_{ij} = c^{tot}. \quad (4)$$

Applying MaxEnt Principle: urban wealth

MaxEnt solution:

$$\mathcal{M}_{ij} = \frac{M_i^e e^{\alpha A_j - \beta c_{ij}}}{Z_i} ,$$

where Z_i is the partition function for location i :

$$Z_i = \sum_j e^{\alpha A_j - \beta c_{ij}} ,$$

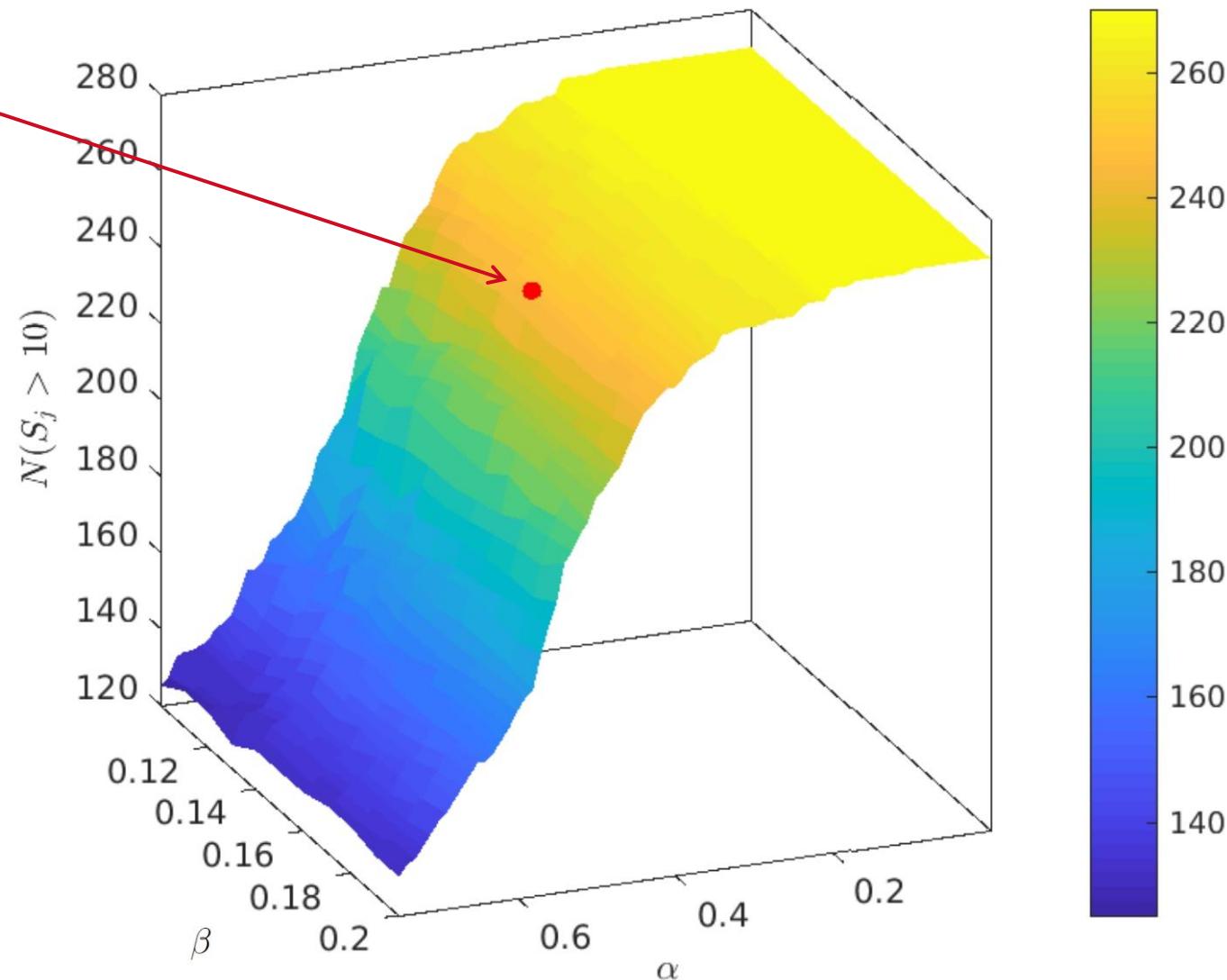
α is the social disposition (preference) to living in a residential area, and β is the travel impedance (reluctance to meet the cost of travel).

$$\frac{dS_j}{dt} = \epsilon([\mathcal{M}_j^p - R_j P_j] - KS_j)$$

- control parameters:
 - social disposition
 - impedance to travel
- order parameter: number of large suburbs

Phase transition in the number of large suburbs

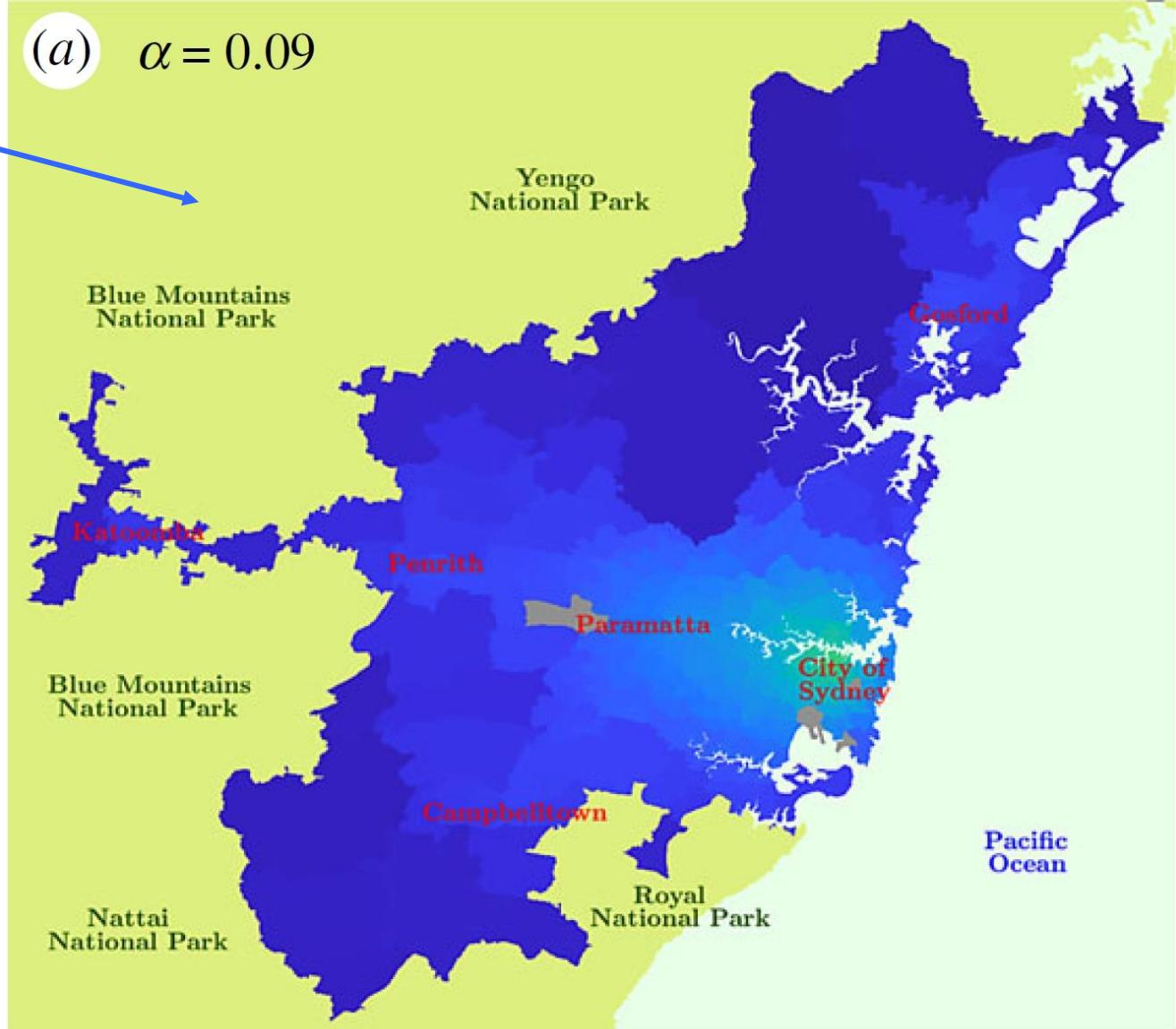
Sydney, 2011-2016





Greater Sydney: monocentric

Low efficiency

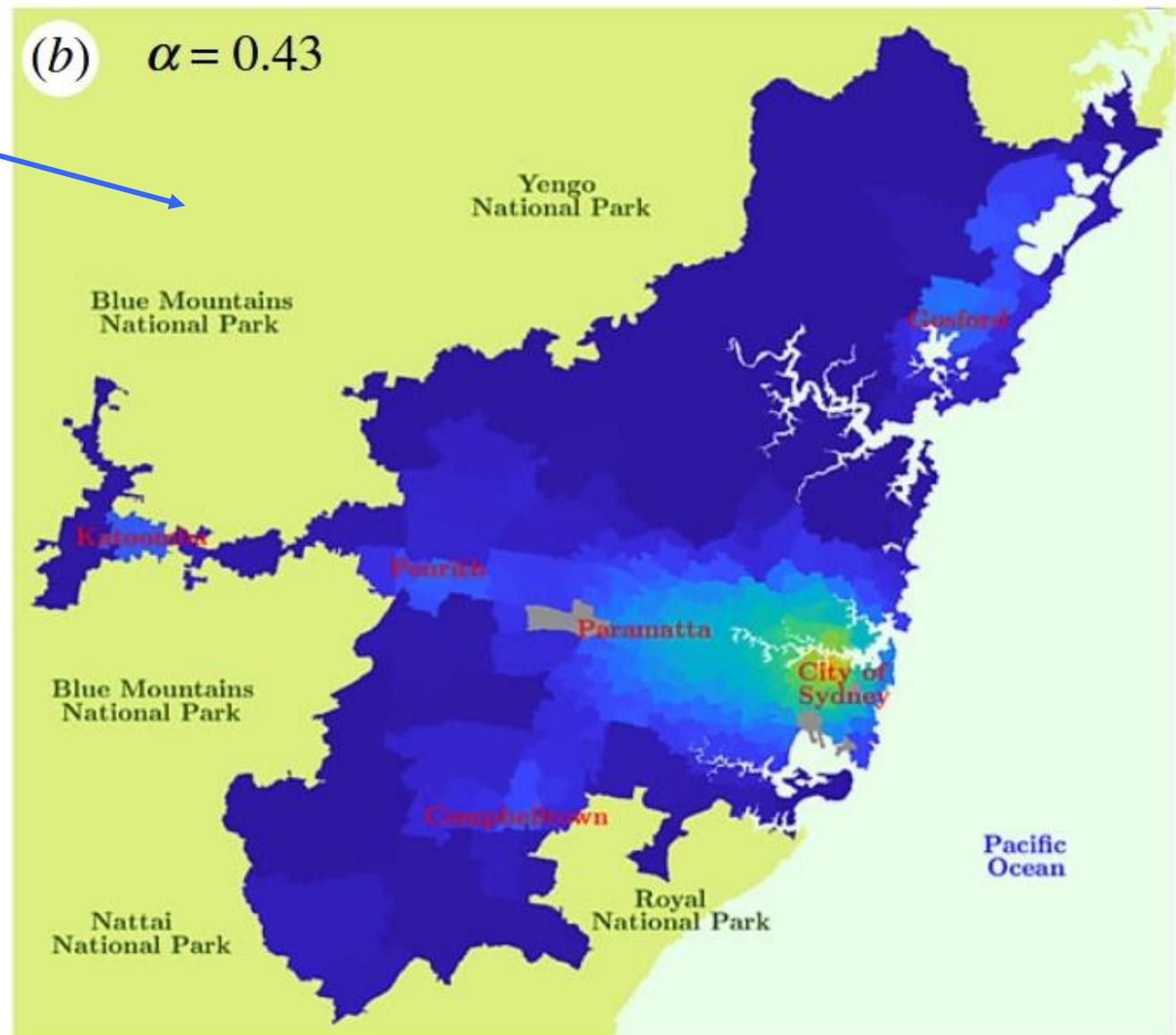




Greater Sydney: (2011-2016)

Medium efficiency

(b) $\alpha = 0.43$

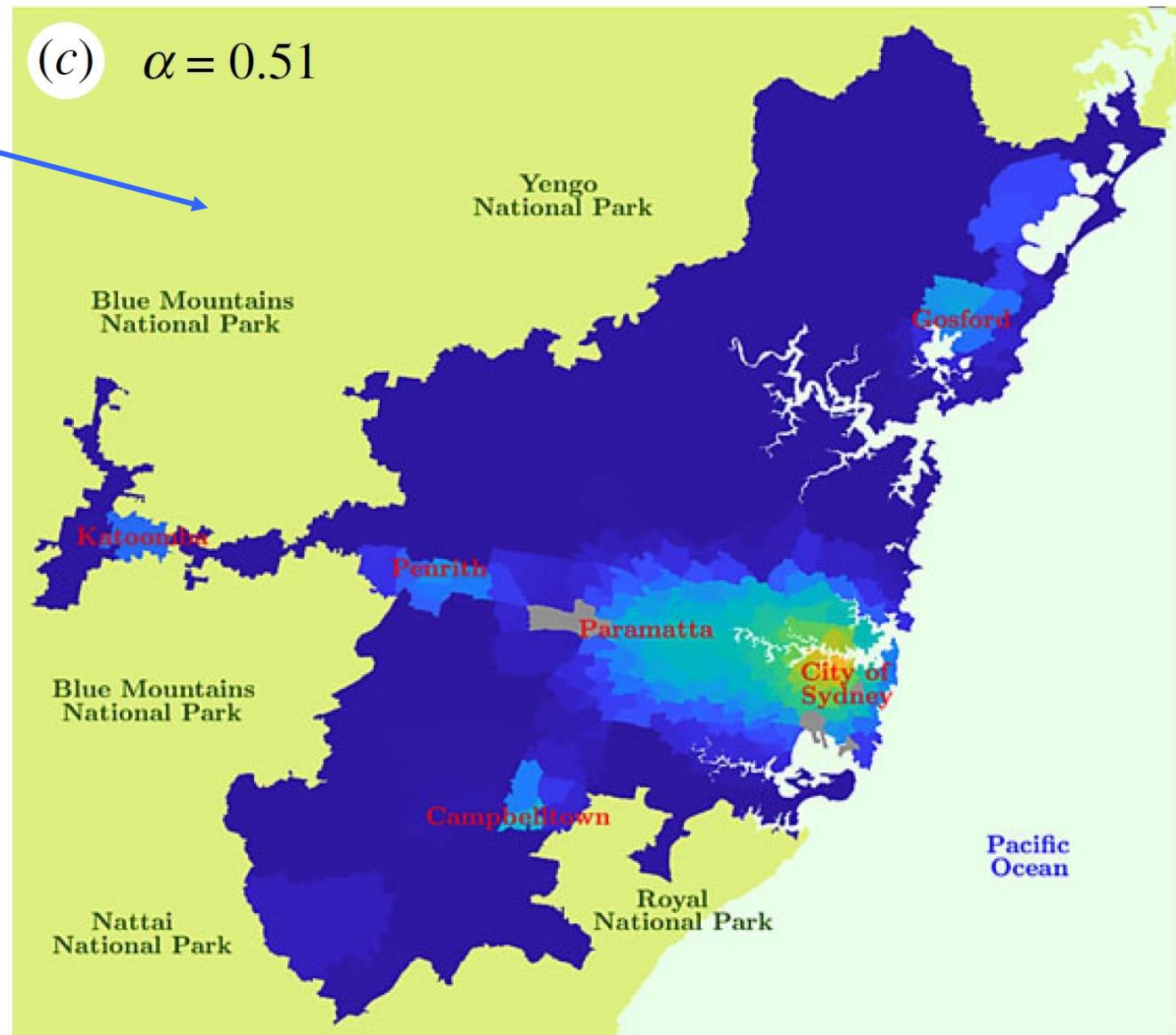




Greater Sydney: possible

Higher efficiency

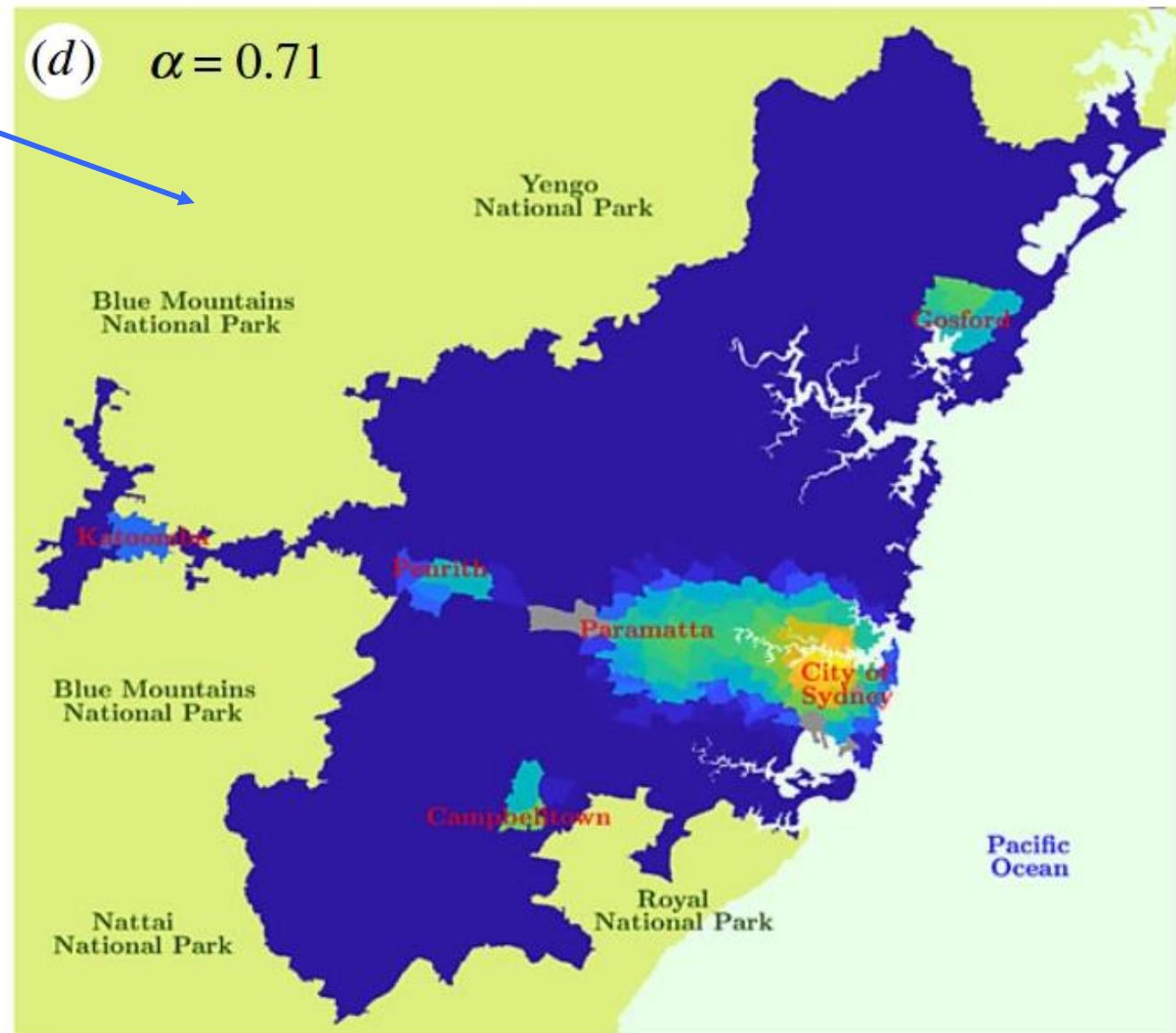
(c) $\alpha = 0.51$



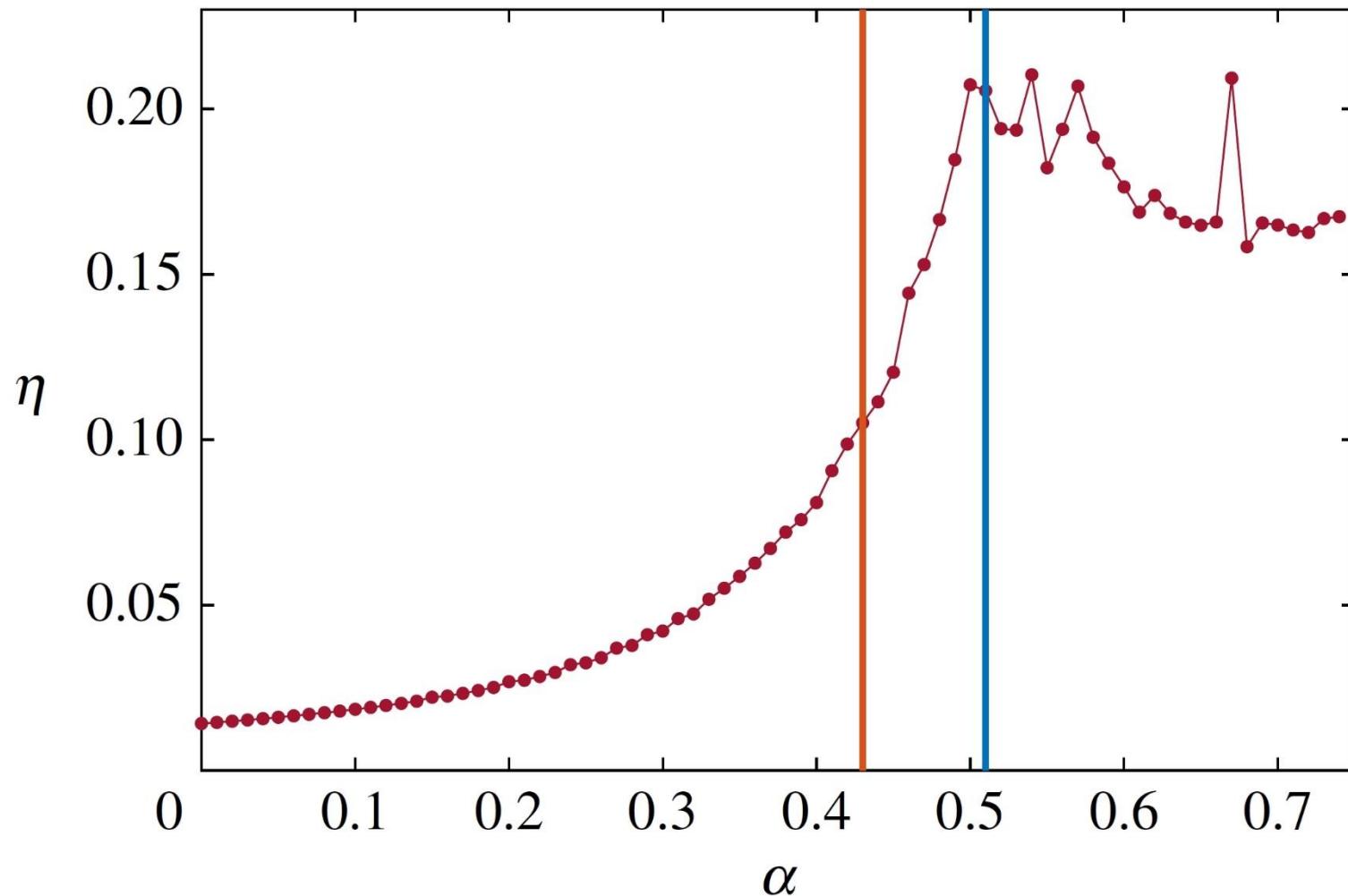


Greater Sydney: polycentric

Medium to high efficiency

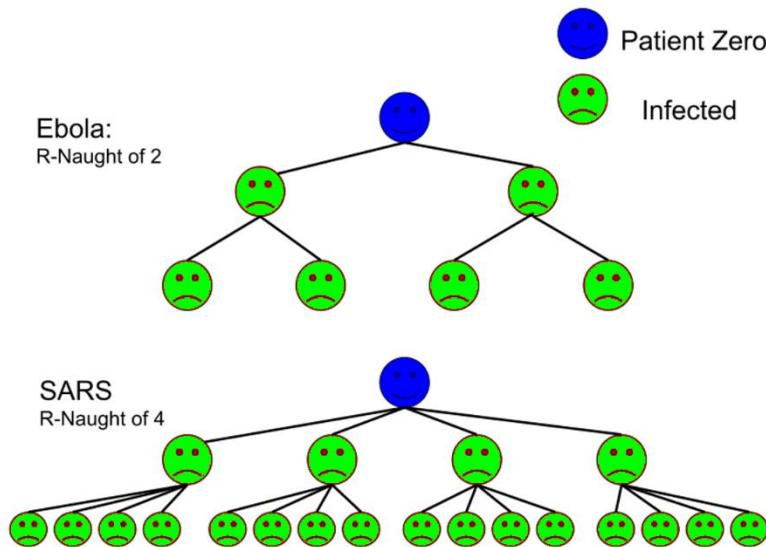


Greater Sydney: thermodynamic efficiency?





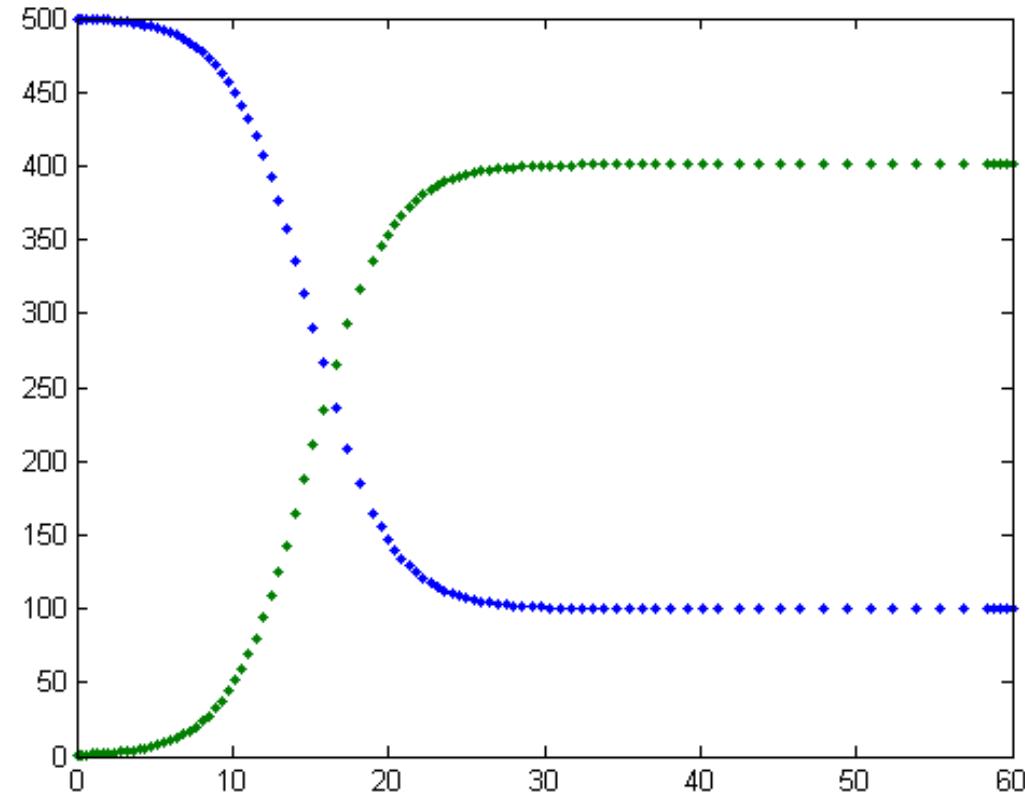
3. Epidemic modelling



$$\frac{dS}{dt} = \gamma I - \beta IS$$
$$\frac{dI}{dt} = \beta IS - \gamma I,$$

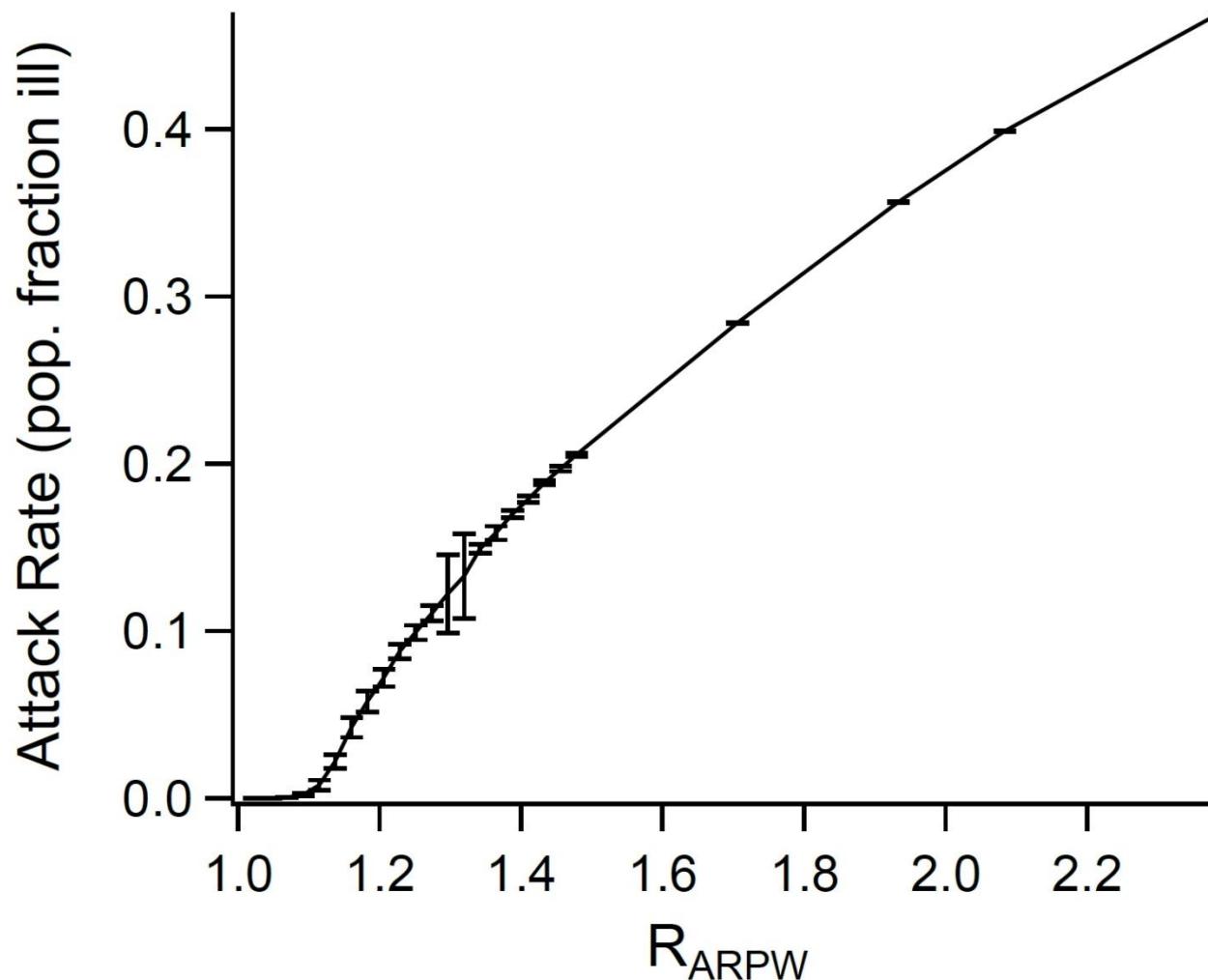
$\beta / \gamma = R_0$

SIS: Susceptible - Infectious - Susceptible





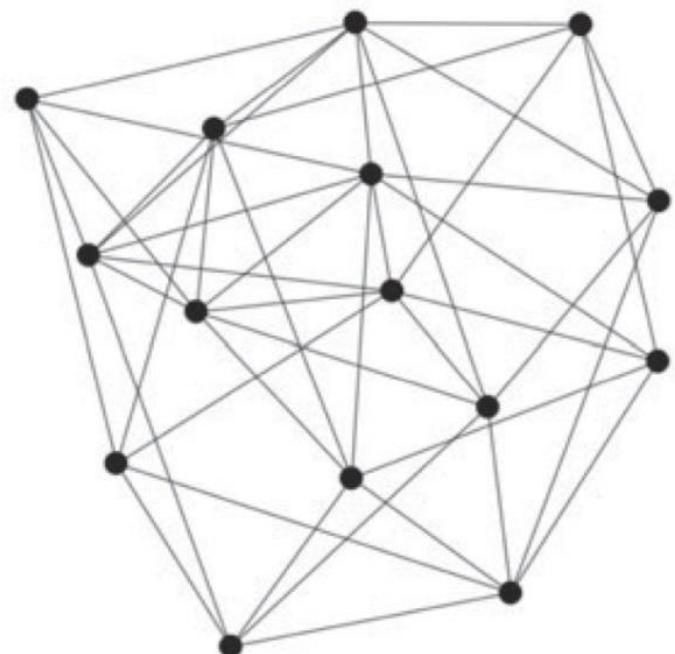
Epidemic threshold: reproductive number R_0



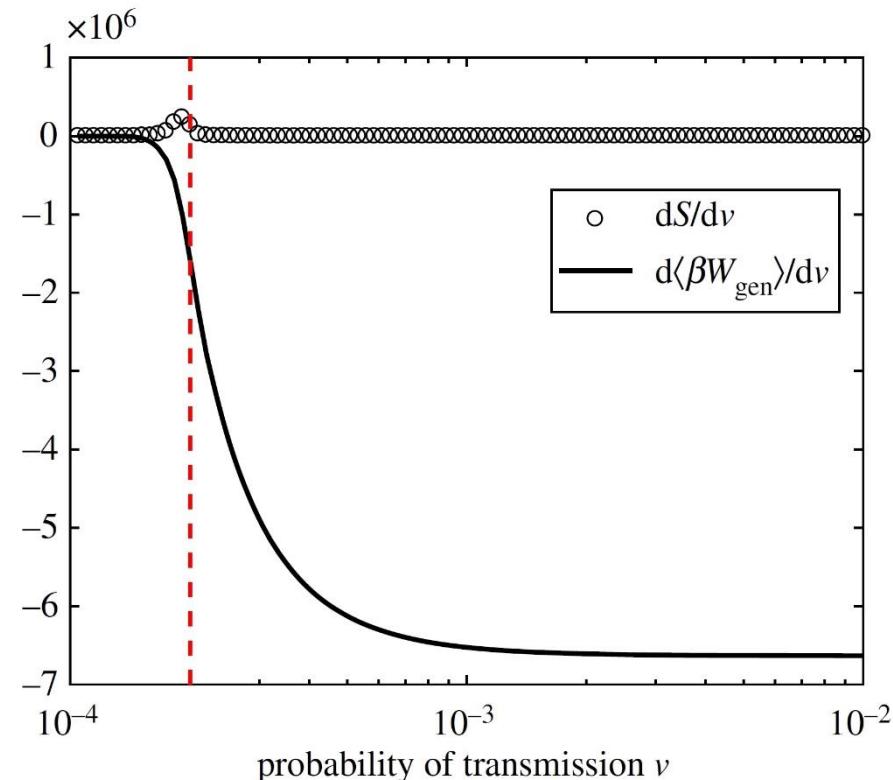
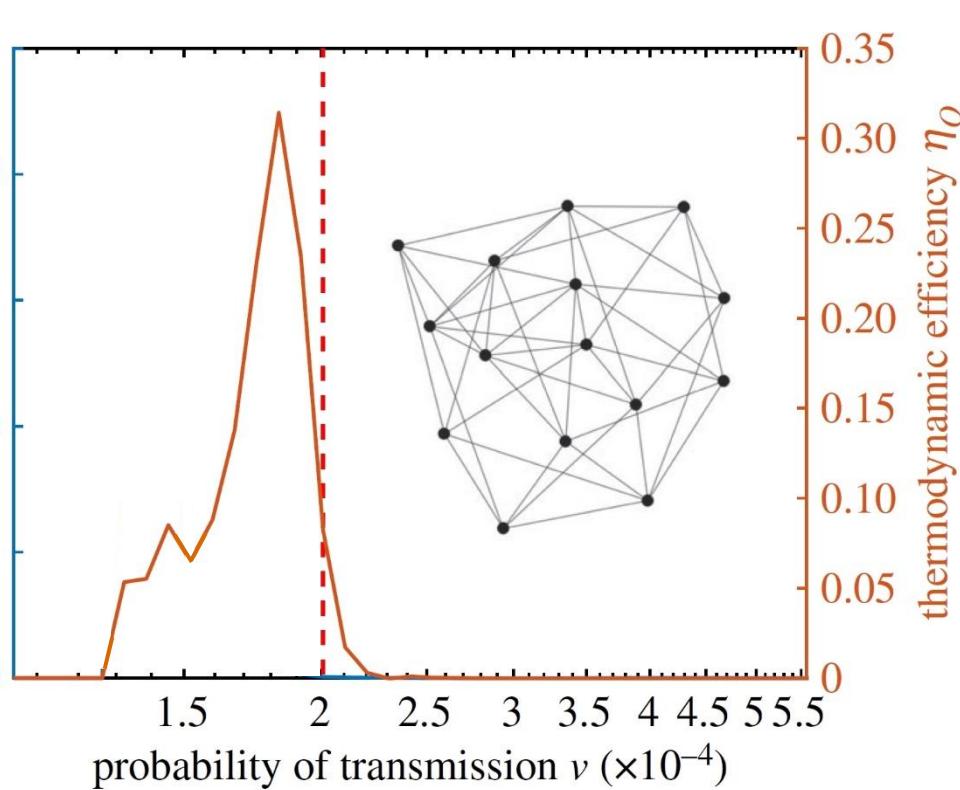


$$\left. \begin{aligned} \frac{dS}{dt} &= \gamma I - \beta IS \\ \frac{dI}{dt} &= \beta IS - \gamma I, \end{aligned} \right\} \quad \begin{aligned} \beta / \gamma &= R_0 \\ \nu & \\ \delta & \end{aligned}$$

$$R_0 = \frac{k\nu}{\nu + \delta - \nu\delta}$$



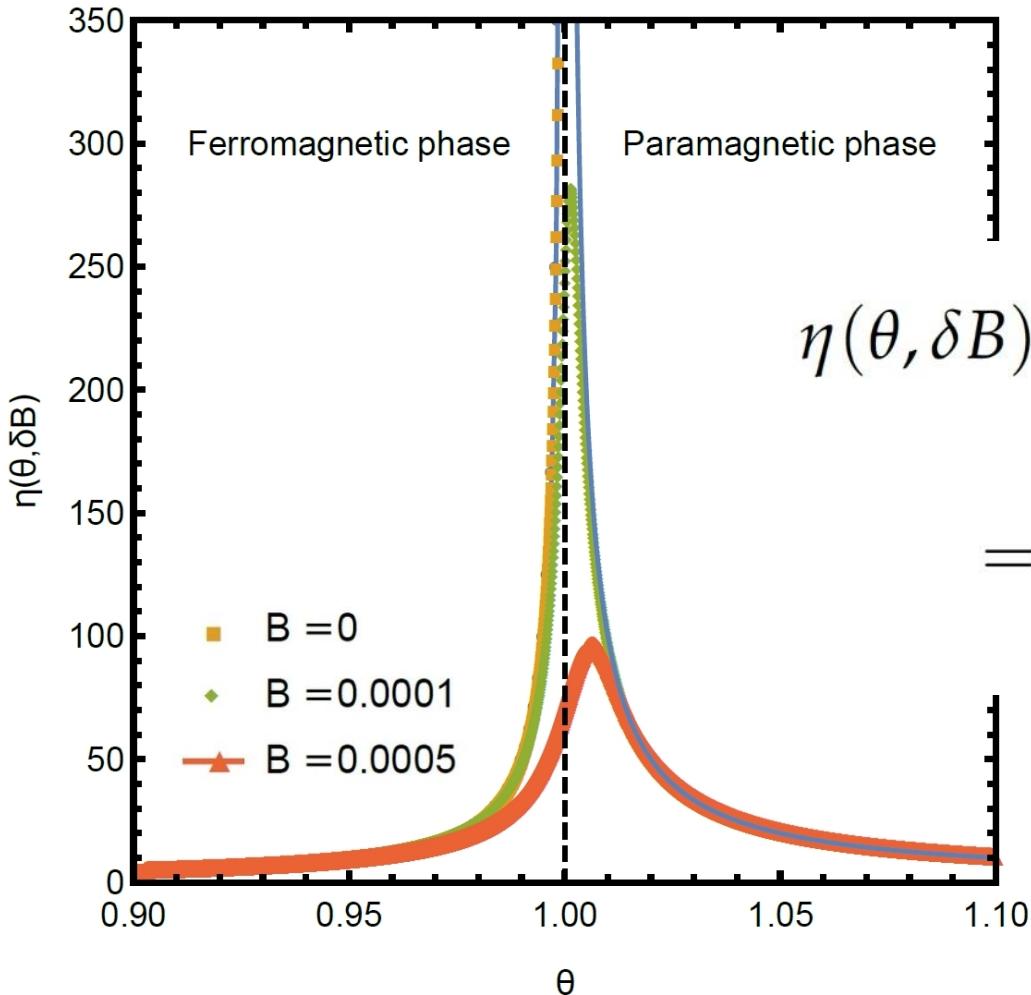
Epidemics as thermodynamic phenomena



- **intervention:** reducing the transmission probability, expending the work
- **pathogen emergence:** increasing the transmission probability, extracting the work



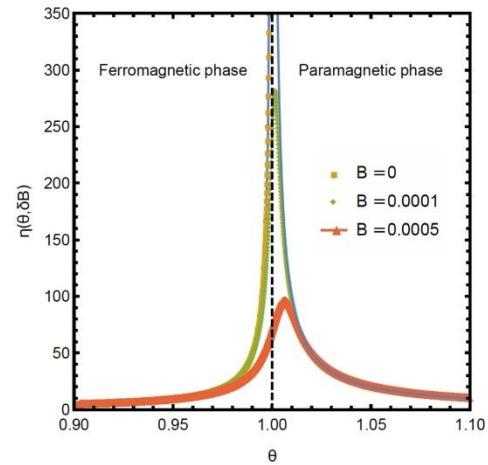
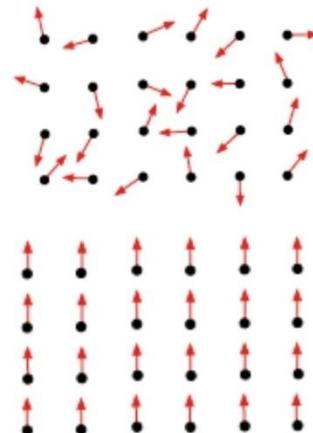
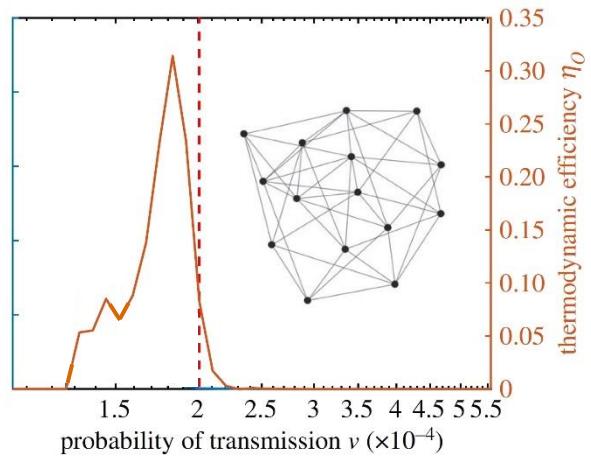
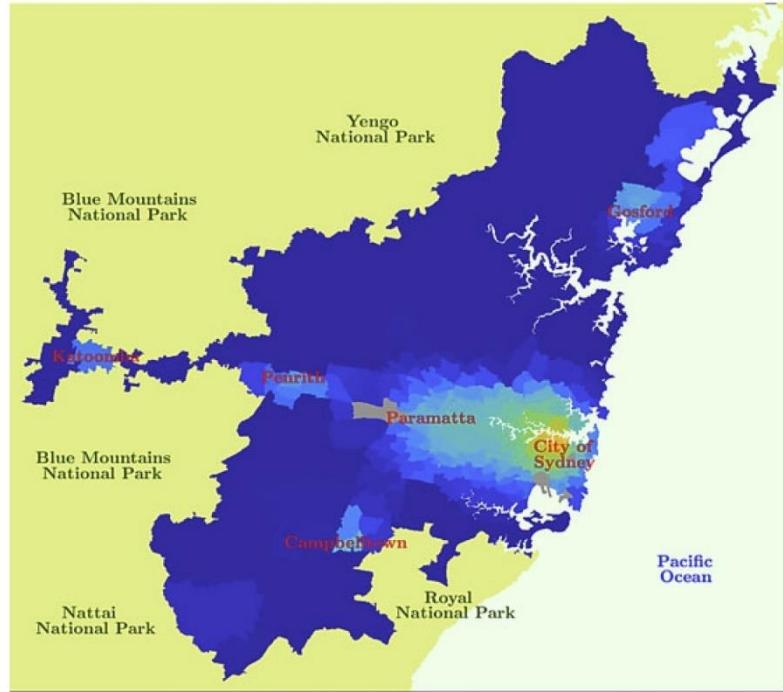
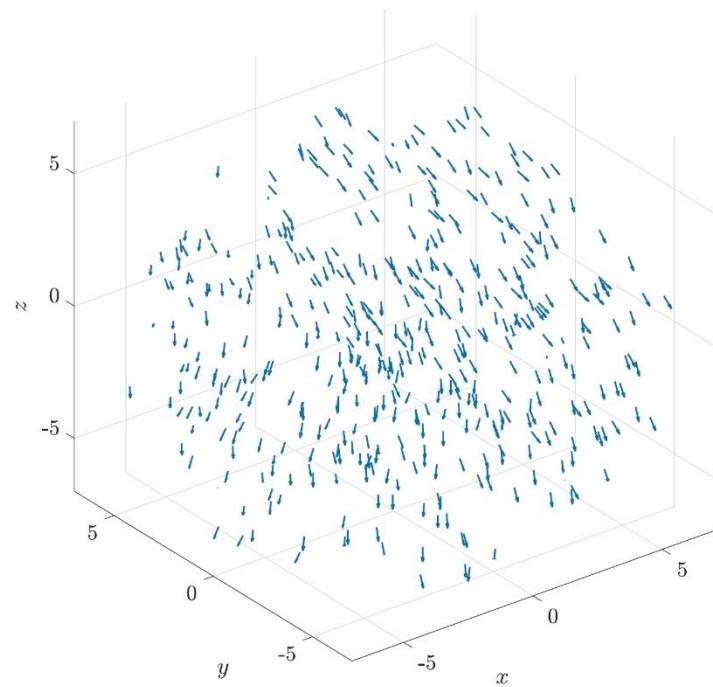
4. Back to magnets...



$$\begin{aligned}\eta(\theta, \delta B) &= \frac{1}{k_B} \frac{\partial s}{\partial B} / \frac{\partial f}{\partial B} \\ &= \begin{cases} -\frac{1}{k_B} \frac{1}{2} t^{-1} & \text{for } t < 0, \\ \frac{1}{k_B \theta_c} t^{-1} & \text{for } t > 0. \end{cases}\end{aligned}$$



Summary (1/2)



- Critical regime: balance between order and chaos
- Thermodynamic and computational perspectives:
 - rate of work carried out to change control parameter = accumulated sensitivity of distributed computation (integral of Fisher information)
- Thermodynamic efficiency:
 - the reduction in uncertainty (the increase in order) from an expenditure of work for a given value of control parameter
 - diverges at critical point for model systems (e.g., Ising model)

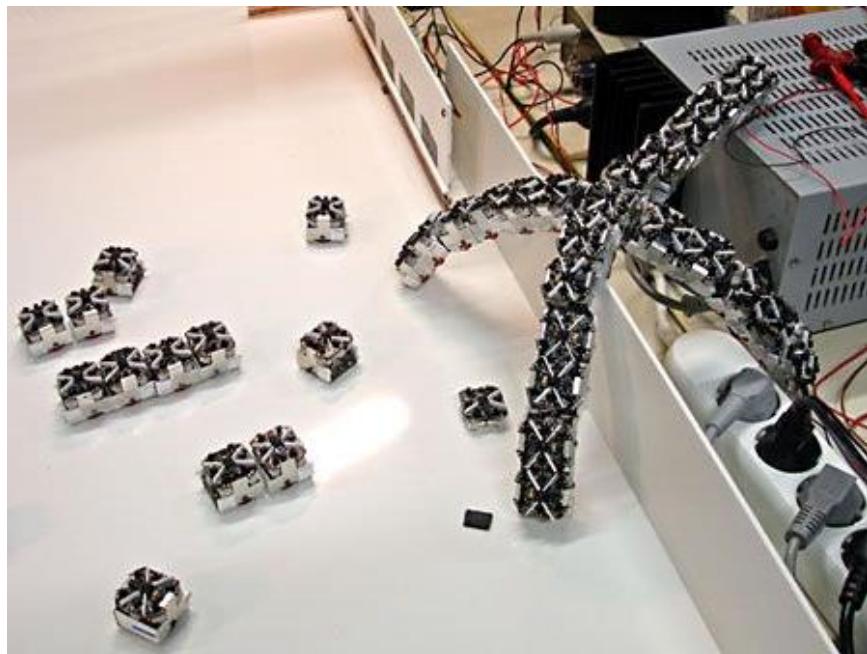
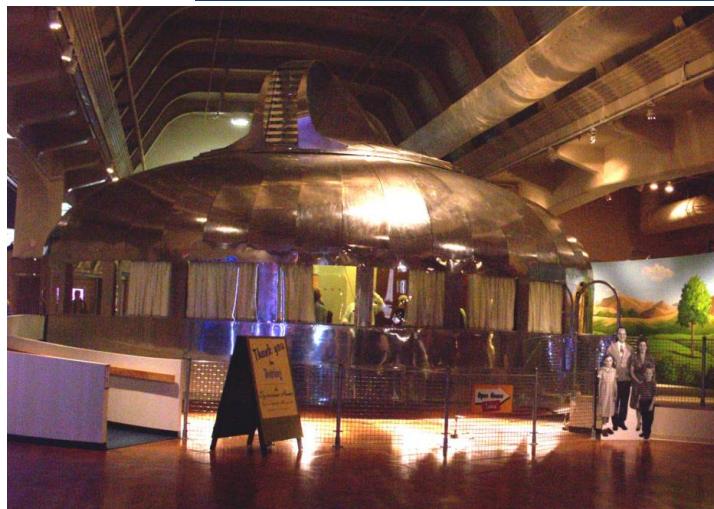
$$\eta(X, \delta X) = -\frac{1}{k_B} \frac{\beta}{|T - T_c|}$$

- *Principle of Super-efficiency:* efficiency of self-organisation is maximal at critical points in dynamical systems



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Dymaxion: ...an example of super-efficiency?



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