The Edge of Chaos and Undecidable Dynamics

Collaborators: Michael Harré, Joseph T. Lizier, Fabio Boschetti, Pavlos Peppas, Stuart Kauffman

Prof. Mikhail Prokopenko

Centre for Complex Systems

Faculty of Engineering

THE UNIVERSITY OF SYDNEY

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Review

Self-referential basis of undecidable dynamics: From the Liar paradox and the halting problem to the edge of chaos

Mikhail Prokopenko [a,*], Michael Harré [a], Joseph Lizier [a], Fabio Boschetti [b], Pavlos Peppas [c,d], Stuart Kauffman [e]

[a] *Centre for Complex Systems, Faculty of Engineering and IT, The University of Sydney, NSW 2006, Australia*
[b] *CSIRO Oceans and Atmosphere, Floreat, WA 6014, Australia*
[c] *Center for AI, School of Software, FEIT, University of Technology Sydney, NSW 2007, Australia*
[d] *Department of Business Administration, University of Patras, Patras 265 00, Greece*
[e] *University of Pennsylvania, Philadelphia, PA 19104, USA*

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Outline

- Edge of chaos, criticality and phase transitions
- Complex systems are dynamical systems with undecidable dynamics
- The Liar paradox and the halting problem
- Self-reference (and diagonalisation)
- Meta-simulation and novelty generation

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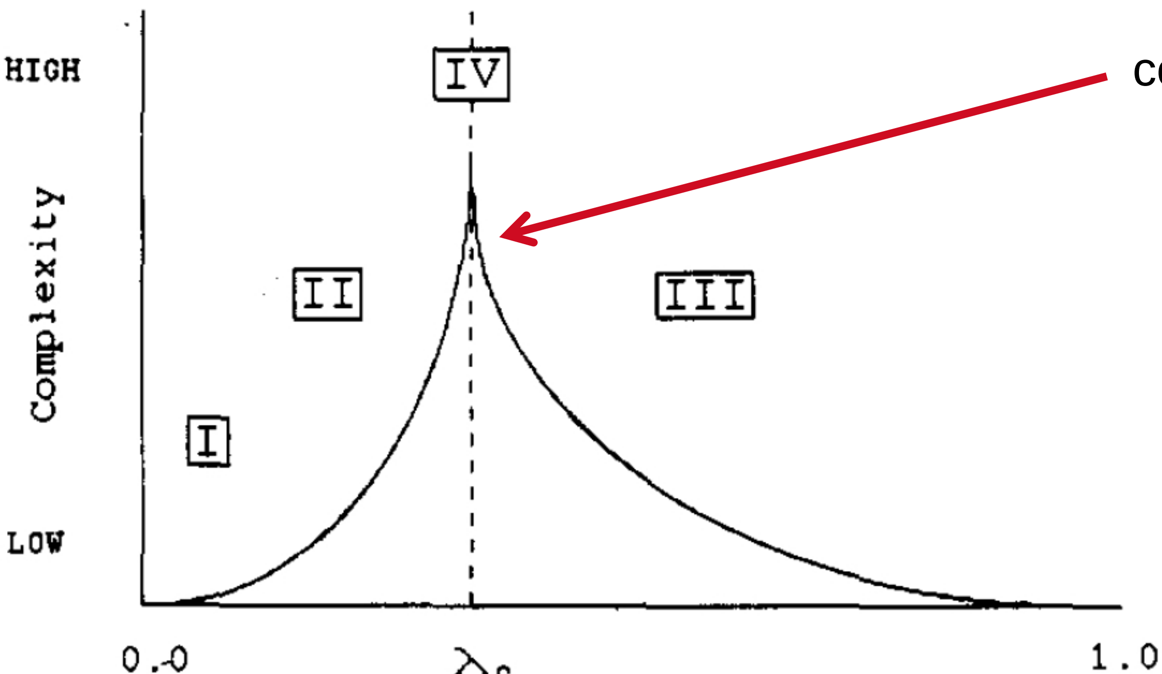
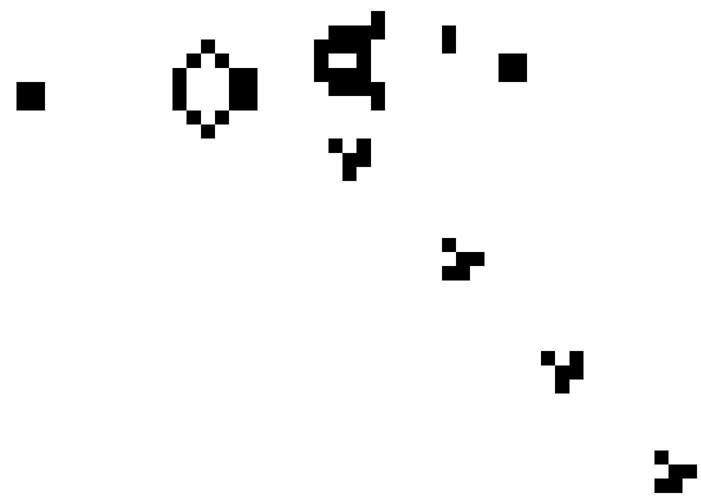
Motivation: dynamics of computation

Chris Langton, "Computation at the edge of chaos: Phase transitions and emergent computation" (1991):

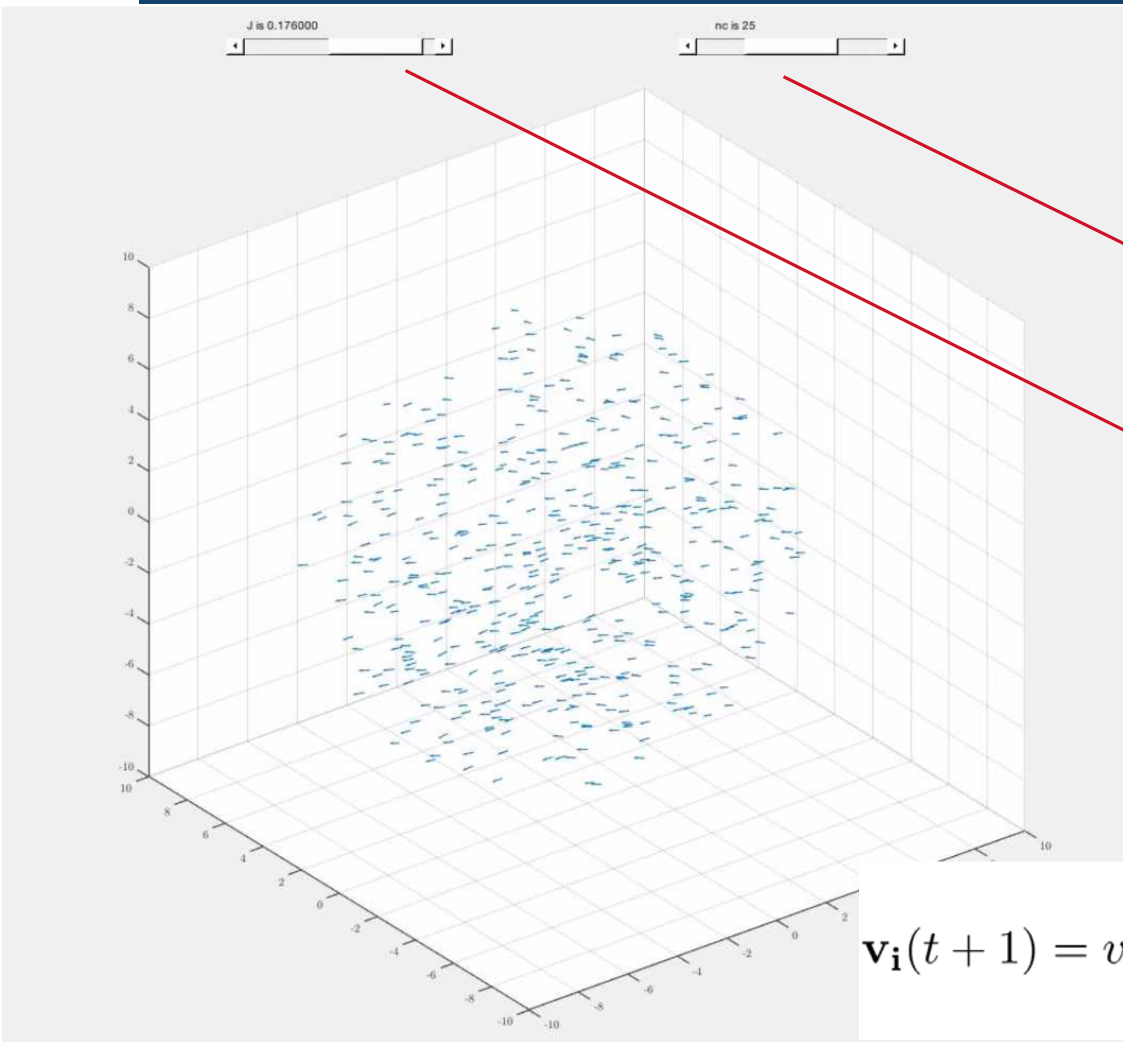
- how can emergence of computation be explained in a *dynamic* setting?
- how is it related to *complexity* of the system in point?



complex high-level structures



Swarming (collective) motion



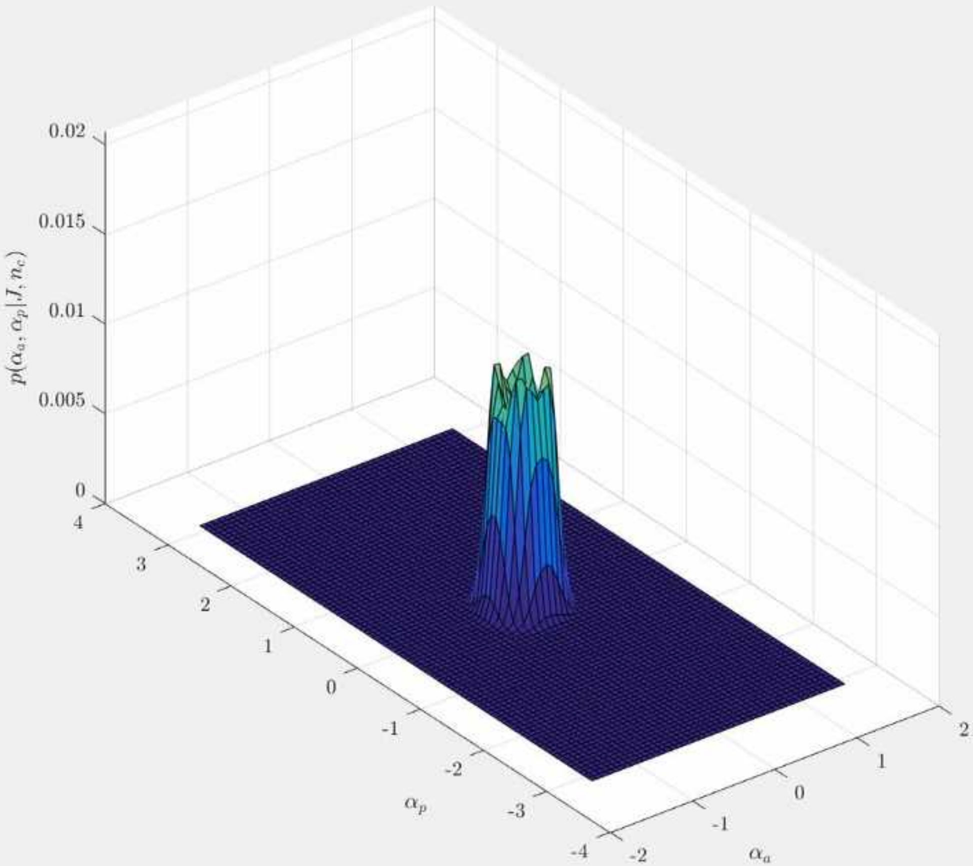
a kinetic phase transition driven by

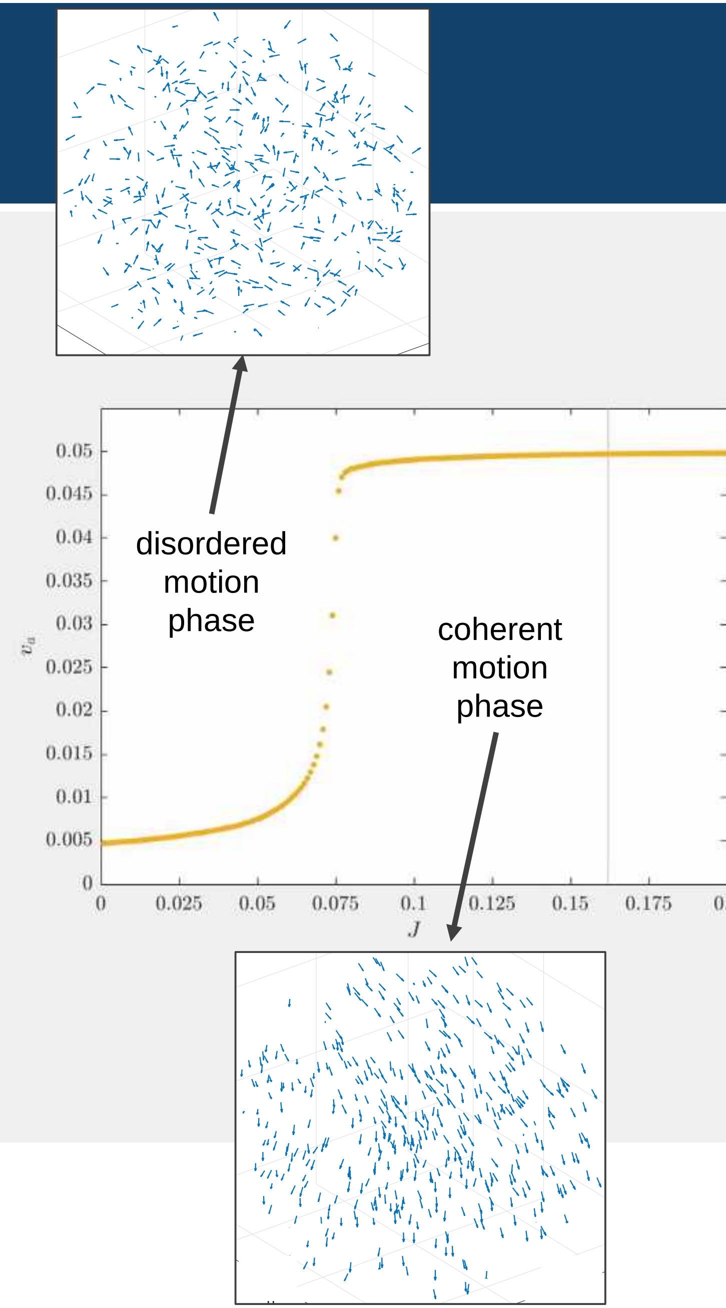
- nearest neighbours N_c
- alignment strength $J = v_0 a$

$$\mathbf{v_i}(t+1) = v_0 \Theta \left[a \sum_{j \in n_c^i} \mathbf{v_j}(t) + b \sum_{j \in n_c^i} f_{ij} + n_c \boldsymbol{\eta}_\mathbf{i} \right]$$

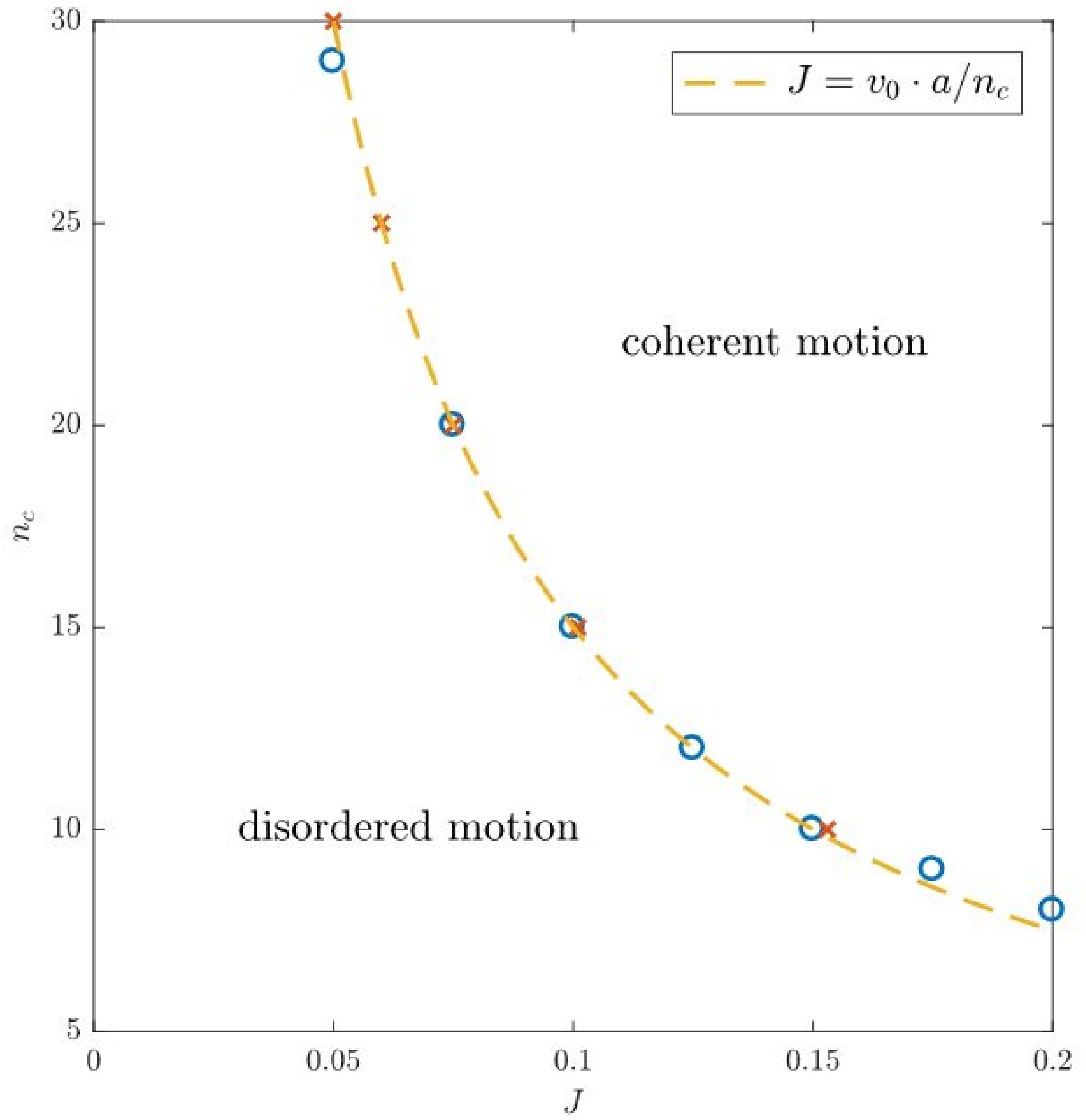
E. Crosato, R. Spinney, R. Nigmatullin, J. T. Lizier, M. Prokopenko, Thermodynamics of collective motion near criticality, *Physical Review E,* 97, 012120, 2018.

The two kinetic phases





Edge of chaos in collective motion



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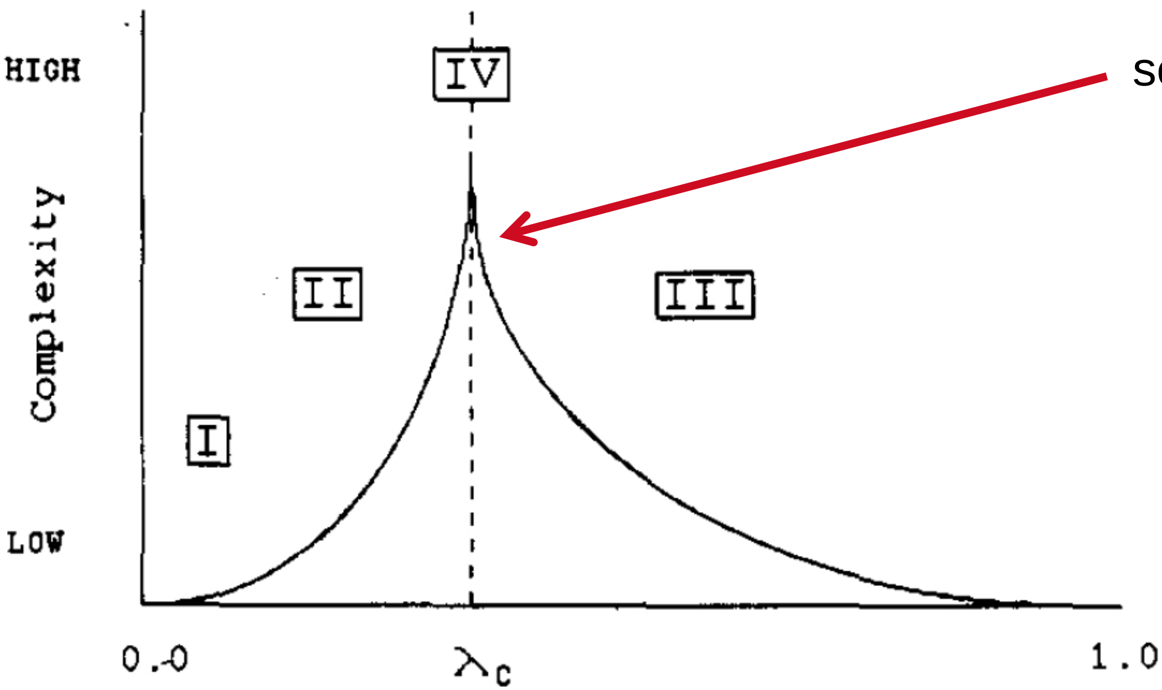
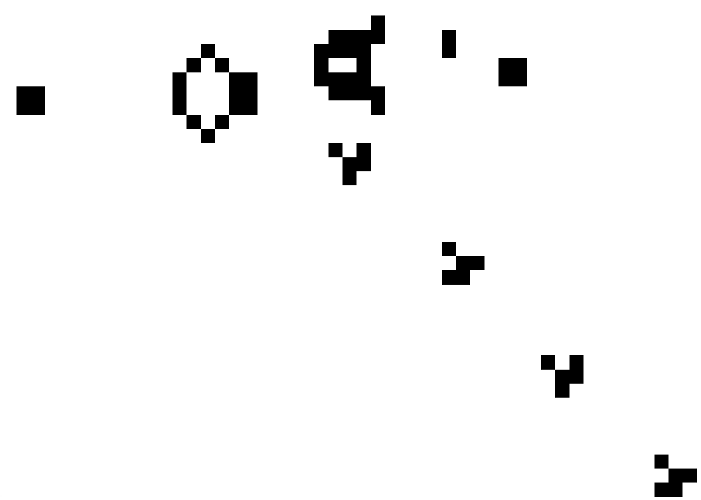
Game of Life

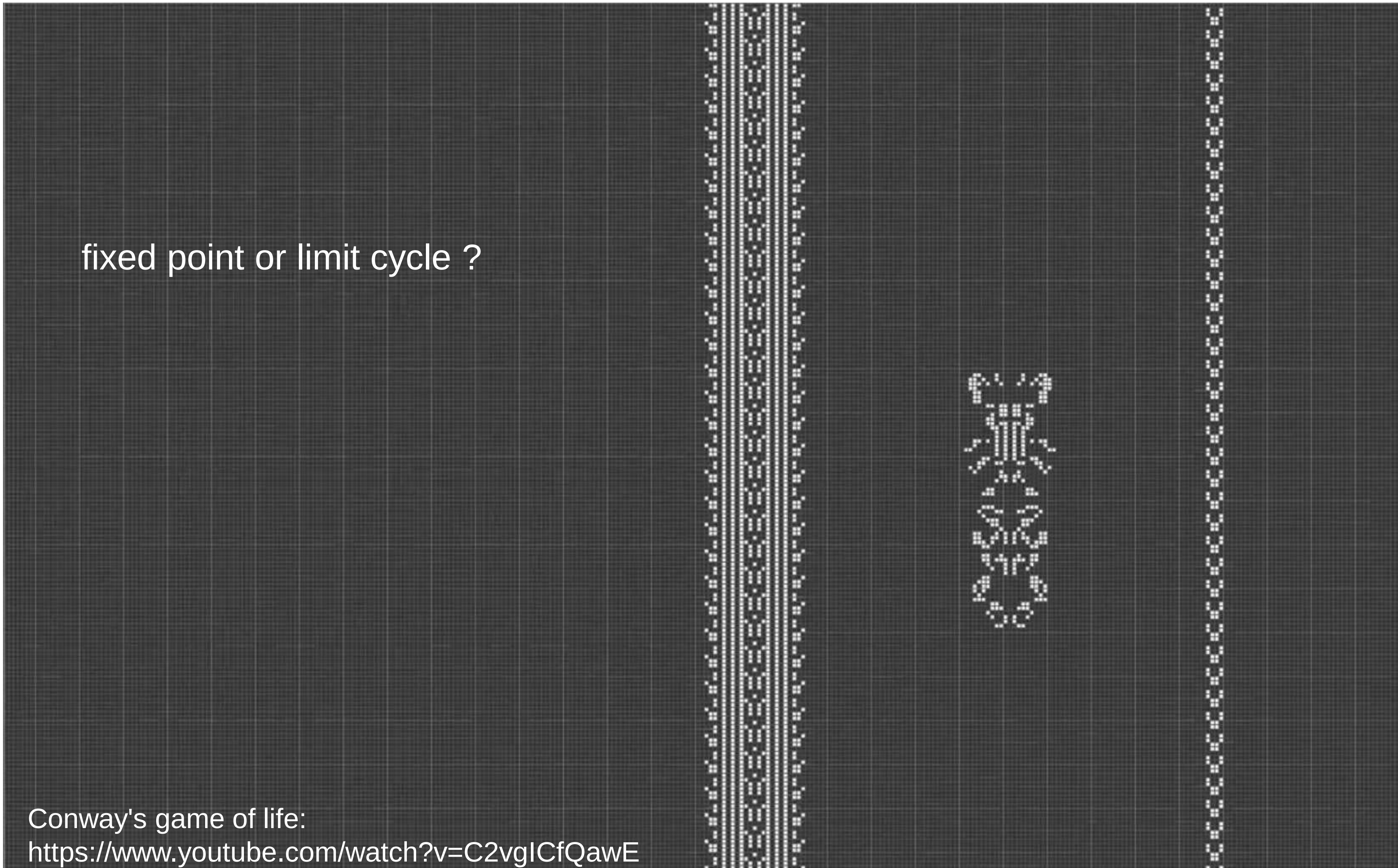
1. Deaths. Any live cell with fewer than two or more than three live neighbours dies.
2. Survivals. Any live cell with two or three live neighbours lives on to the next generation.
3. Births. Any dead cell with exactly three live neighbours becomes a live cell.



self-organising structures



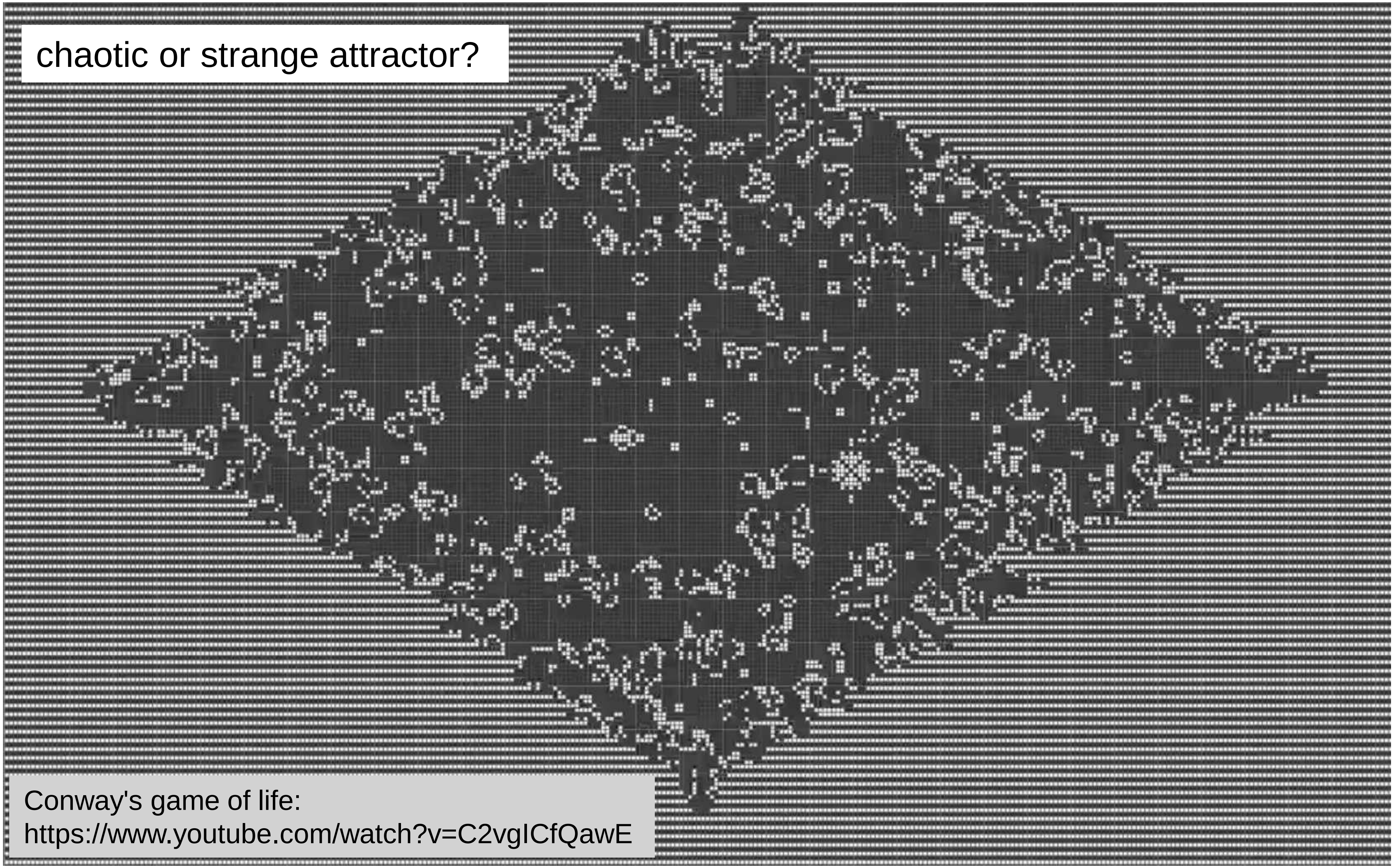
Game of Life: convergence?



fixed point or limit cycle ?

Conway's game of life:
https://www.youtube.com/watch?v=C2vgICfQawE

Game of Life: convergence?



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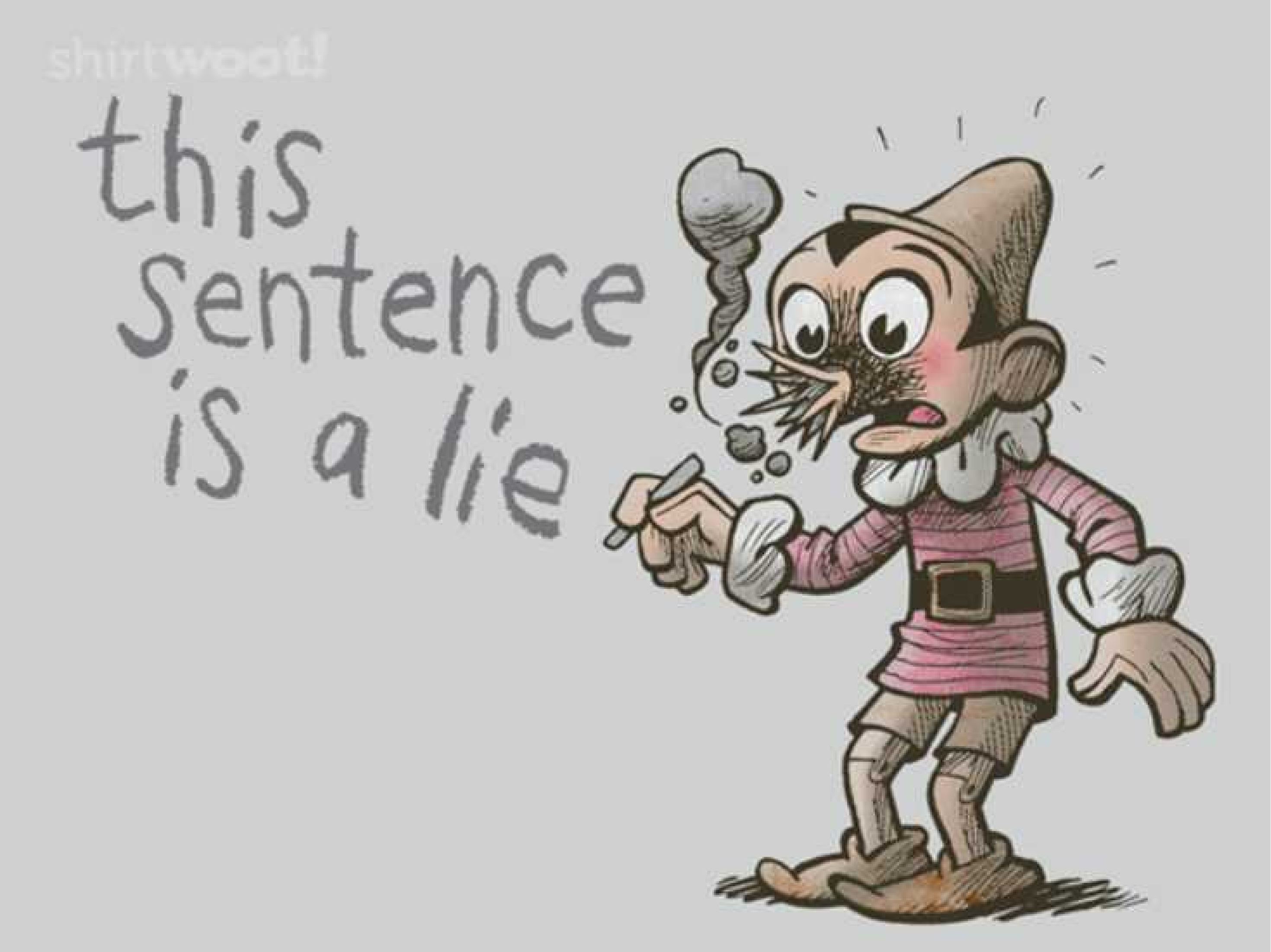
The Liar paradox

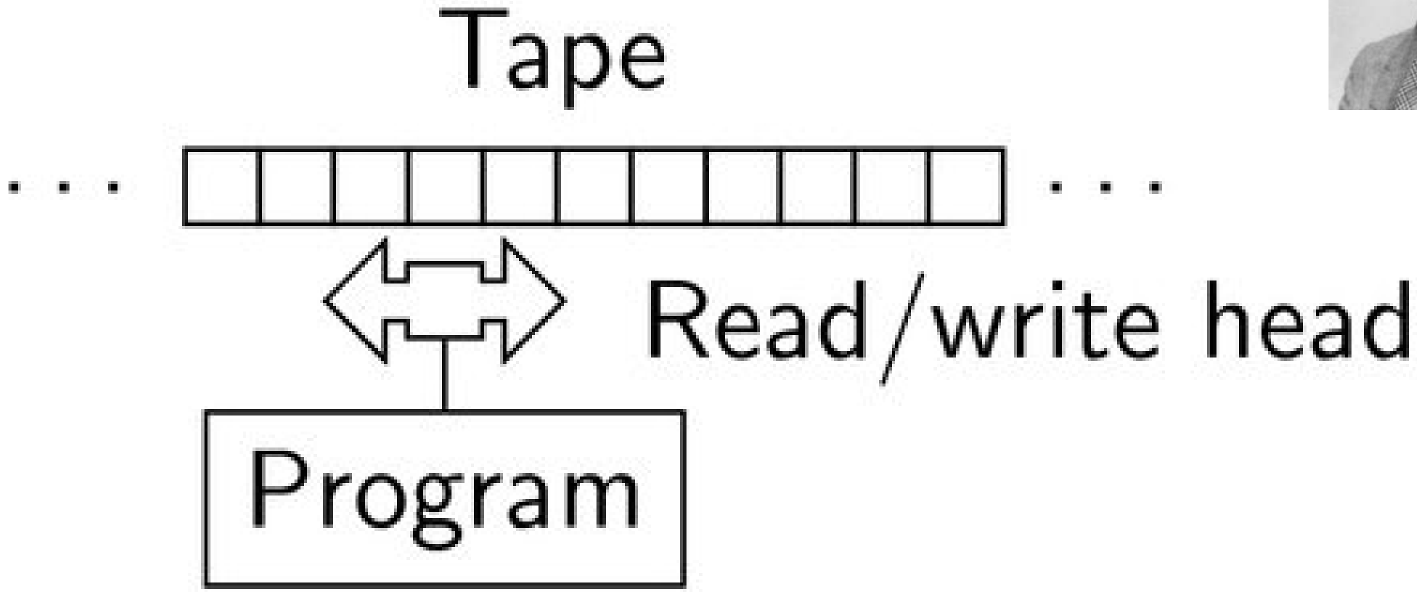


https://shirt.woot.com/offers/liars-paradox

Turing Machine (…not that one)

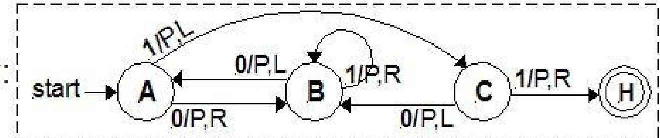
The Imitation Game, 2014

Turing Machine



Turing Machine

3-state busy beaver:



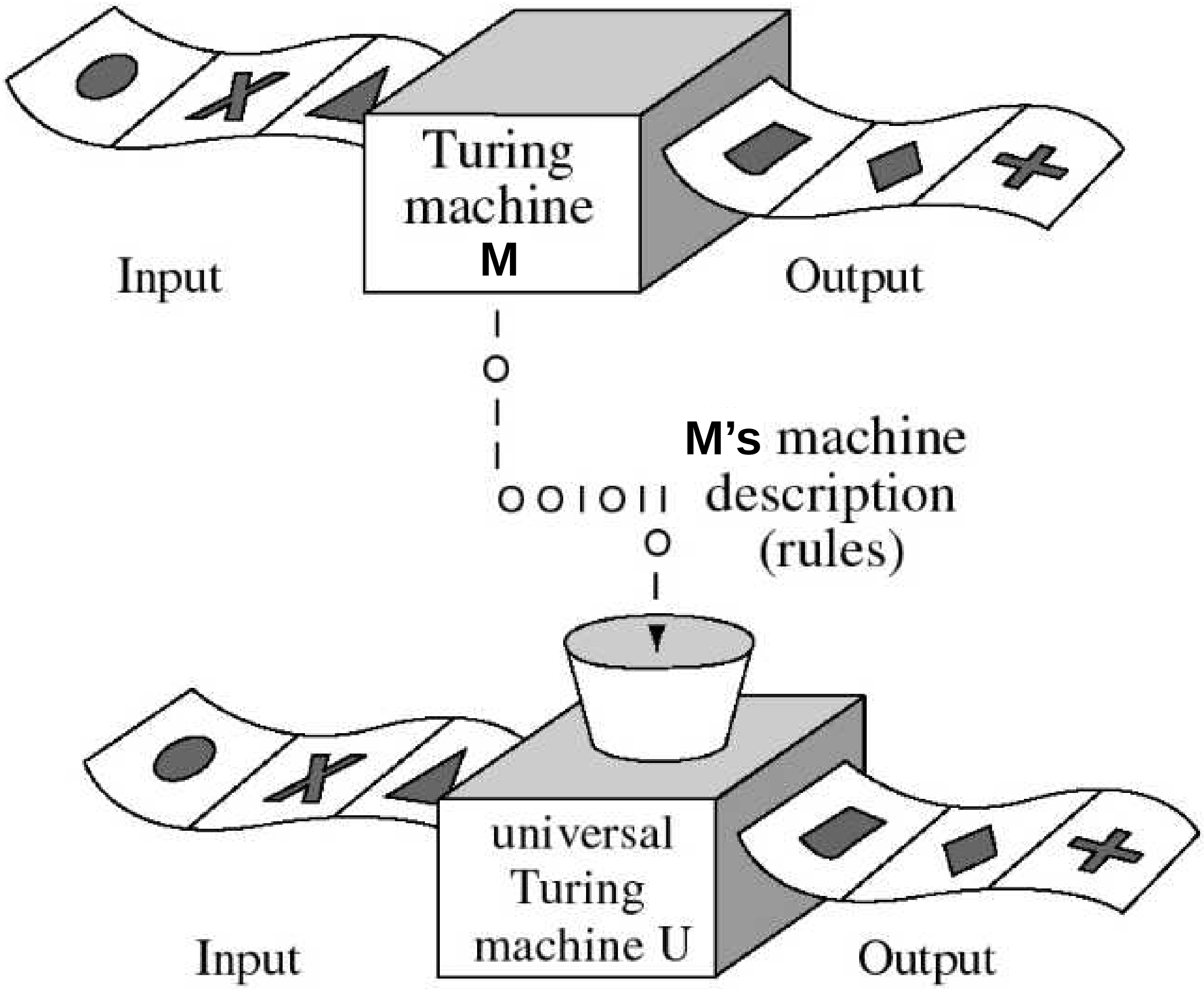
Sequence	Instruction							Head									Instruction: A	B	C	H	Total system state -- complete configuration (aka "instantaneous description") TAPE & TABLE & HEAD
		0	0	0	0	0	0	0	<u>0</u>	0	0	0	0	0	0	0					
1	A	0	0	0	0	0	0	0	<u>0</u>	0	0	0	0	0	0	0	A				A <u>0</u>
2	B	0	0	0	0	0	0	0	<u>0</u>	1	0	0	0	0	0	0		B			B <u>0</u> 1
3	A	0	0	0	0	0	0	1	<u>1</u>	0	0	0	0	0	0	0	A				1 A <u>1</u>
4	C	0	0	0	0	0	1	1	<u>0</u>	0	0	0	0	0	0	0			C		1 1 C <u>0</u>
5	B	0	0	0	0	1	1	1	<u>0</u>	0	0	0	0	0	0	0		B			1 1 1 B <u>0</u>
6	A	0	0	0	1	1	1	1	<u>0</u>	0	0	0	0	0	0	0	A				1 1 1 1 A <u>0</u>
7	B	0	0	0	0	1	1	1	<u>1</u>	1	0	0	0	0	0	0		B			1 1 1 B <u>1</u> 1
8	B	0	0	0	0	0	1	1	<u>1</u>	1	1	0	0	0	0	0		B			1 1 B <u>1</u> 1 1
9	B	0	0	0	0	0	0	1	<u>1</u>	1	1	0	0	0	0	0		B			1 B <u>1</u> 1 1 1
10	B	0	0	0	0	0	0	0	<u>1</u>	1	1	1	1	0	0	0		B			B <u>1</u> 1 1 1 1
11	B	0	0	0	0	0	0	0	<u>0</u>	1	1	1	1	1	0	0		B			B <u>0</u> 1 1 1 1 1
12	A	0	0	0	0	0	0	1	<u>1</u>	1	1	1	1	0	0	0	A				1 A <u>1</u> 1 1 1 1
13	C	0	0	0	0	0	1	1	<u>1</u>	1	1	1	0	0	0	0			C		1 1 C <u>1</u> 1 1 1
14	H	0	0	0	0	0	0	1	<u>1</u>	1	1	1	1	0	0	0				H	1 H <u>1</u> 1 1 1 1

time -->

Progress of the computation (state-trajectory) of a 3-state busy beaver

Wikipedia

Universal Turing Machine: can simulate any other machine



Glenn D. Blank et al., 2003

http://www.cse.lehigh.edu/~glennb/um/1intro.pdf

Universal Decider Turing Machine: does it exist?

$$P([M, w]) = \begin{cases} accept & \text{if } M \text{ accepts } w \\ reject & \text{if } M \text{ does not accept } w \end{cases}$$

M. Prokopenko, M. Harré, J. Lizier, F. Boschetti, P. Peppas, S. Kauffman, Self-referential basis of undecidable dynamics: from The Liar Paradox and the halting Problem to the edge of chaos, *Physics of Life Reviews*, 31: 134-156, 2019.

Universal Decider Turing Machine: does it exist?

$$P([M, w]) = \begin{cases} accept & \text{if } M \text{ accepts } w \\ reject & \text{if } M \text{ does not accept } w \end{cases}$$

$$V([M]) = \begin{cases} reject & \text{if } M \text{ accepts } [M] \\ accept & \text{if } M \text{ does not accept } [M] \end{cases}$$

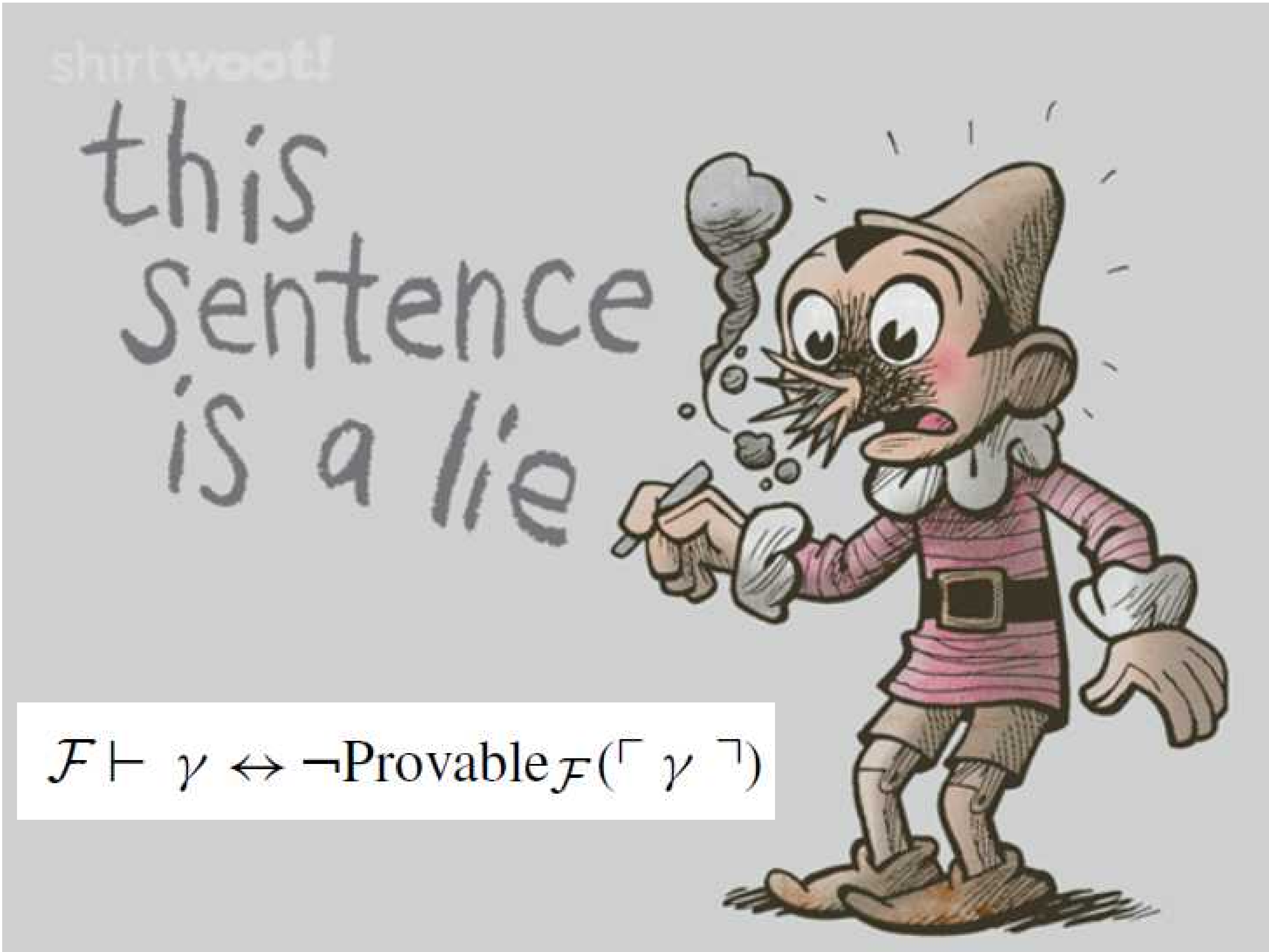
Universal Decider Turing Machine: does not exist!

$$P([M,w]) = \begin{cases} accept & \text{if } M \text{ accepts } w \\ reject & \text{if } M \text{ does not accept } w \end{cases}$$

$$V([M]) = \begin{cases} reject & \text{if } M \text{ accepts } [M] \\ accept & \text{if } M \text{ does not accept } [M] \end{cases}$$

$$V([V]) = \begin{cases} reject & \text{if } V \text{ accepts } [V] \\ accept & \text{if } V \text{ does not accept } [V] \end{cases}$$

The Liar paradox



$$\mathcal{F} \vdash \gamma \leftrightarrow \neg \mathrm{Provable}_{\mathcal{F}}(\ulcorner \gamma \urcorner)$$

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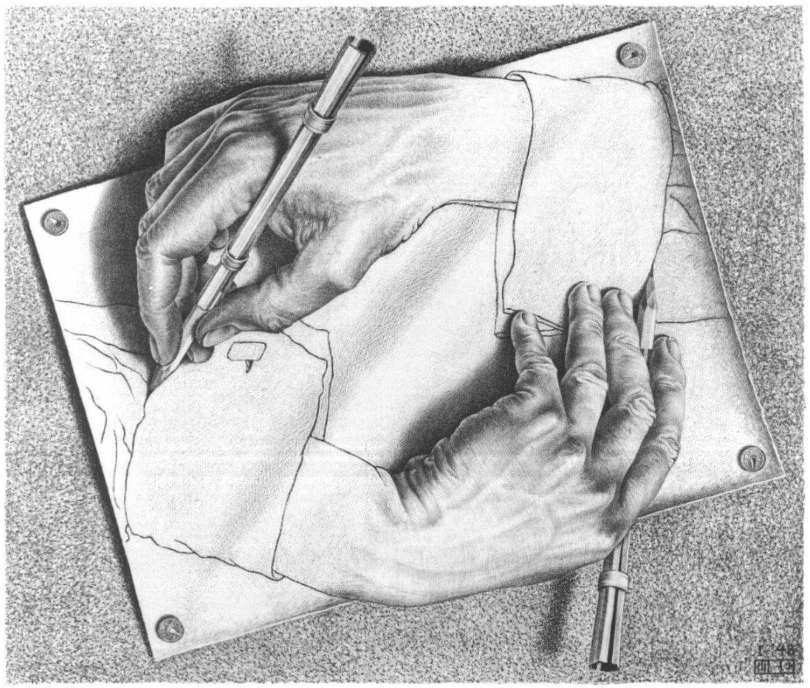
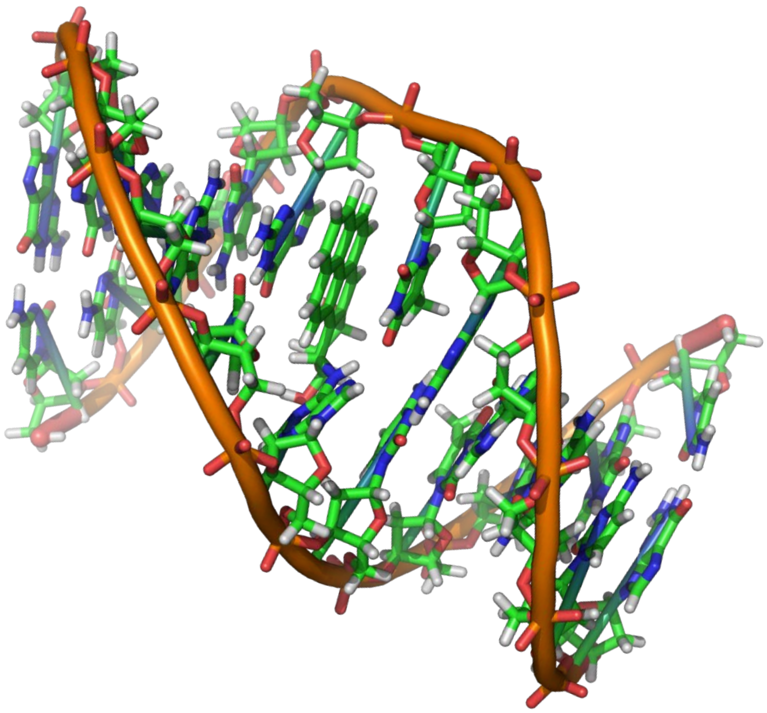
Self-reference



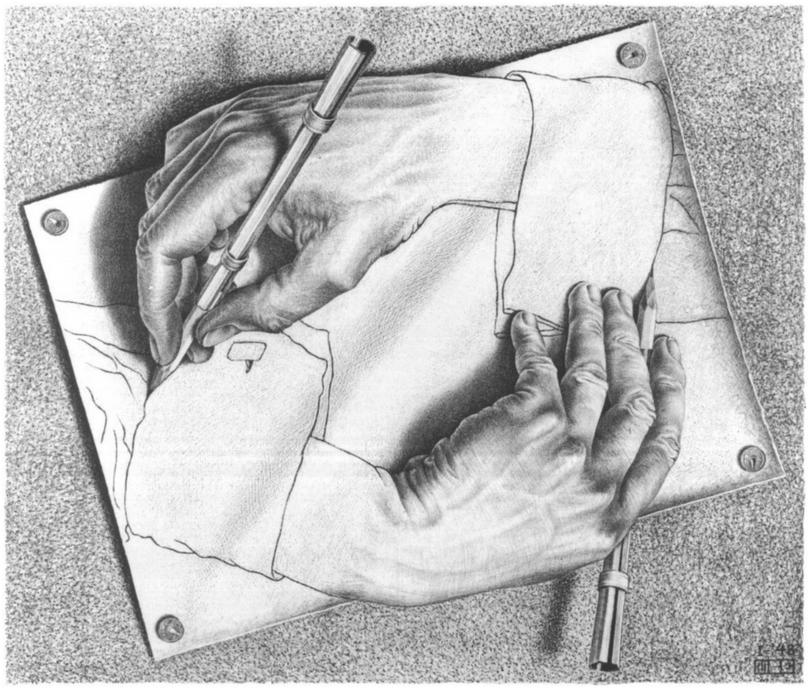
Drawing Hands:

the product, the hands, are undertaking the operation – the drawing of the hands



DNA: genetic instructions (*sequence*) used in development and functioning of a living organism (*structure*) – a set of “blueprints” needed to construct other components of cells, and copy itself

Self-reference



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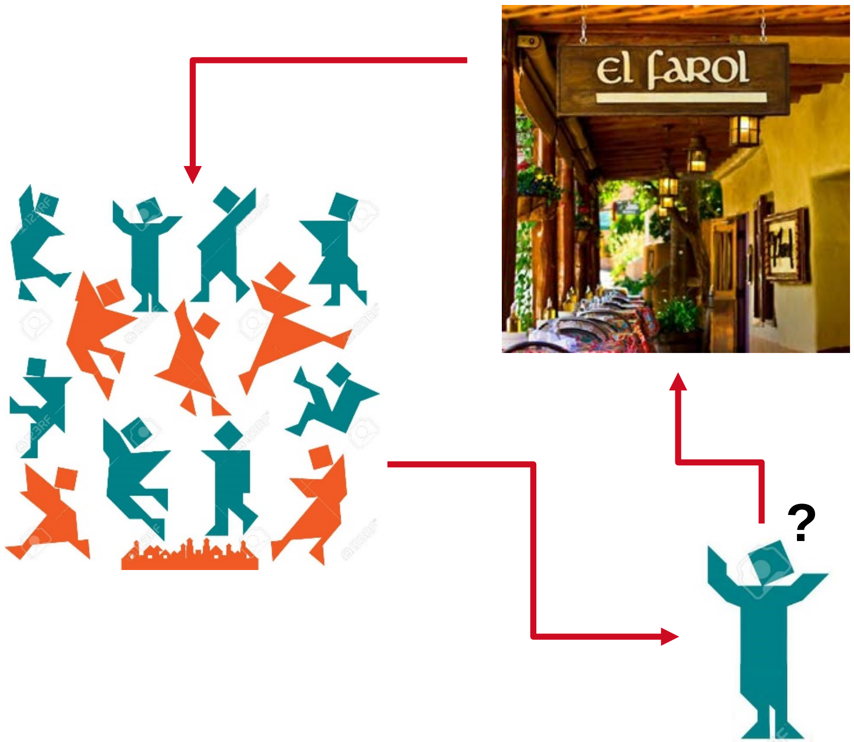

El Farol Bar Problem:

if less than 60% of the population go to the bar, then 🙂🙂🙂🙂

if more than 60% of the population go to the bar, then 🙁🙁🙁🙁🙁🙁🙁🙁

Self-reference and diagonalisation

Table 3
The cell i, j is 'accept' if M_i accepts $[M_j]$

	$[M_1]$	$[M_2]$	$[M_3]$	$\cdots$
M_1	accept		accept	
M_2	accept	accept	accept	$\cdots$
M_3		accept		
$\vdots$		$\vdots$		$\ddots$

M. Prokopenko, M. Harré, J. Lizier, F. Boschetti, P. Peppas, S. Kauffman, Self-referential basis of undecidable dynamics: from The Liar Paradox and the halting Problem to the edge of chaos, *Physics of Life Reviews*, 31: 134-156, 2019.

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M_2	accept	accept	accept	$\cdots$
M_3		accept		
$\vdots$		$\vdots$		$\ddots$

universal decider

Table 4
The cell i, j is the outcome of running P on $[M_i[M_j]]$.

	$[M_1]$	$[M_2]$	$[M_3]$	$\cdots$
M_1	accept	reject	accept	
M_2	accept	accept	accept	$\cdots$
M_3	reject	accept	reject	
$\vdots$		$\vdots$		$\ddots$

Self-reference and diagonalisation

inverter of universal decider

Table 5
The cell (k, j) is the outcome of running $V = M_k$ (the inverter of P) on $[M_k[M_j]]$

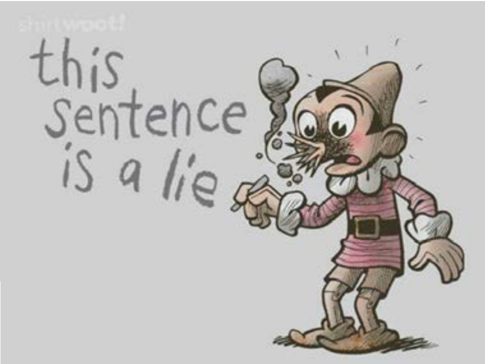
	$[M_1]$	$[M_2]$	$[M_3]$	$\cdots$	$[M_k]$	$\cdots$
M_1	accept	reject	accept		accept	
M_2	accept	accept	accept	$\cdots$	reject	$\cdots$
M_3	reject	accept	reject		reject	
$\vdots$		$\vdots$		$\ddots$		
M_k	reject	reject	accept			
$\vdots$		$\vdots$				$\ddots$

Self-reference and diagonalisation

inverter of universal decider

Table 5
The cell (k, j) is the outcome of running $V = M_k$ (the inverter of P) on $[M_k[M_j]]$. A contradiction occurs at cell (k, k).

	$[M_1]$	$[M_2]$	$[M_3]$	$\cdots$	$[M_k]$	$\cdots$
M_1	accept	reject	accept		accept	
M_2	accept	accept	accept	$\cdots$	reject	$\cdots$
M_3	reject	accept	reject		reject	
$\vdots$		$\vdots$		$\ddots$		
M_k	reject	reject	accept		?	
$\vdots$		$\vdots$				



$$\mathcal{F} \vdash \gamma \leftrightarrow \neg\mathrm{Provable}_{\mathcal{F}}(\ulcorner \gamma \urcorner)$$

Self-reference and diagonalisation

inverter of universal decider

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	$[M_1]$	$[M_2]$	$[M_3]$	$\cdots$	$[M_k]$	$\cdots$
M_1	accept	reject	accept		accept	
M_2	accept	accept	accept	$\cdots$	reject	$\cdots$
M_3	reject	accept	reject		reject	
$\vdots$		$\vdots$		$\ddots$		
M_k	reject	reject	accept		?	
$\vdots$		$\vdots$				

$$\mathcal{F} \vdash \gamma \leftrightarrow \neg \mathrm{Provable}_{\mathcal{F}}(\ulcorner \gamma \urcorner)$$

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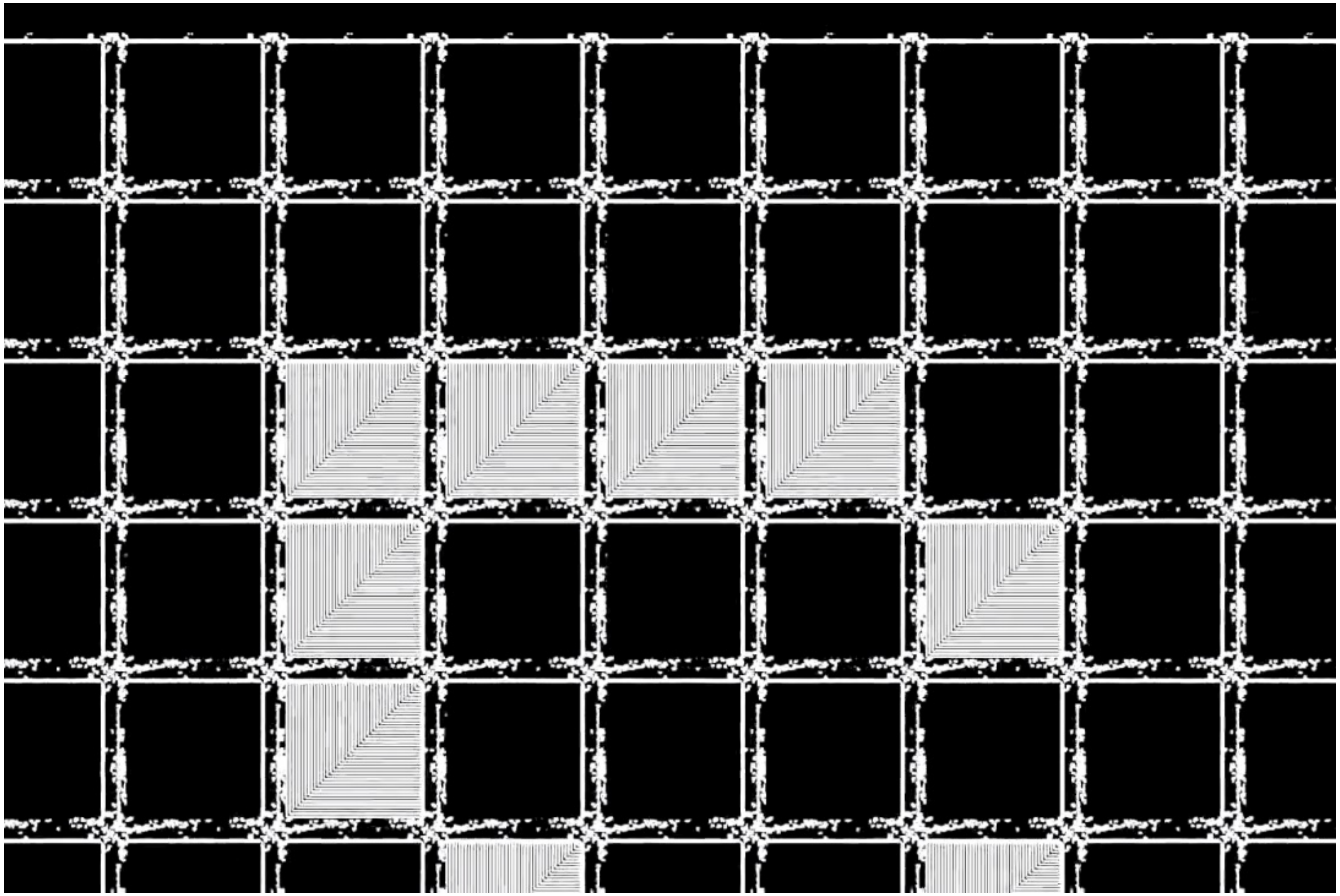
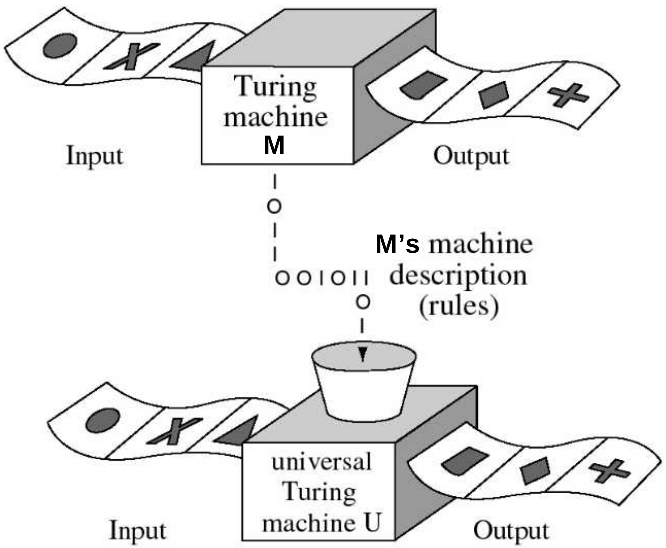
Universal Cellular Automata: a Metapixel

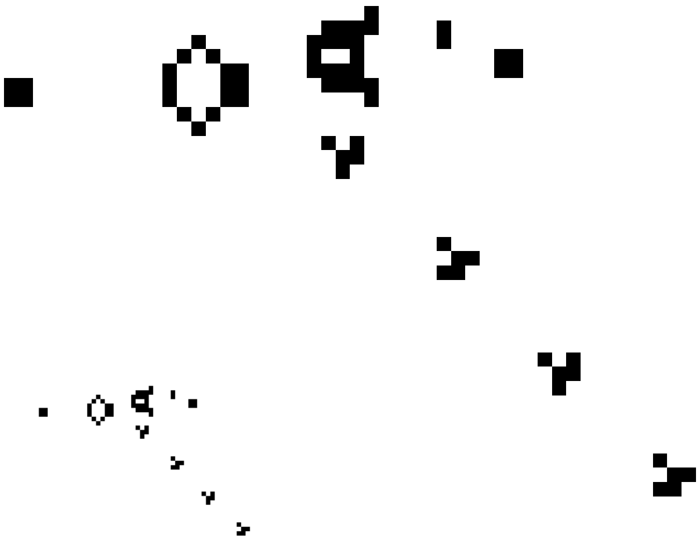


“Life in Life” by Phillip Bradbury: https://www.youtube .com /watch ?time _continue =4 &v =xP5 -iIeKXE8

Conclusions

$$\mathcal{F} \vdash \gamma \leftrightarrow \neg\mathrm{Provable}_{\mathcal{F}}(\ulcorner \gamma \urcorner)$$





Conclusions

- recursive formal systems, Turing machines and Cellular Automata are deeply related
- these frameworks can produce universal computation and generate undecidable dynamics
- undecidability is generated by self-reference, infinite computation and negation
- computational novelty can be created by agents using the diagonalization argument
- complex systems are dynamical systems with undecidable dynamics

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