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Local assortativity and growth of Internet

M. Piraveenan[1,2], M. Prokopenko[1,a], and A.Y. Zomaya[2]

[1] CSIRO Information and Communications Technology Centre, Locked Bag 17, North Ryde, NSW 1670, Australia
[2] School of Information Technologies, The University of Sydney, NSW 2006, Australia

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Abstract. Local assortativity has been recently proposed as a measure to analyse complex networks. It has been noted that the Internet Autonomous System level networks show a markedly different local assortativity profile to most biological and social networks. In this paper we show that, even though several Internet growth models exist, none of them produce the local assortativity profile that can be observed in the real AS networks. We introduce a new generic growth model which can produce a linear local assortativity profile similar to that of the Internet. We verify that this model accurately depicts the local assortativity profile criteria of Internet, while also satisfactorily modelling other attributes of AS networks already explained by existing models.

PACS. 89.20.Hh World Wide Web, Internet – 89.75.Hc Networks and genealogical trees – 89.75.Da Systems obeying scaling laws – 89.75.Fb Structures and organization in complex systems

1 Introduction

In the past decade, multiple large-scale complex networks have been analysed in terms of their global topology and local structure [1–8]. One network that has been given much attention is the Internet Autonomous System (AS) level network, where each node represents an autonomous system present in the Internet and the edges represent a commercial agreement between two Internet Service Provides (who own the two ASs). Such an agreement defines whether they agree to exchange data traffic and how to charge each other. As such, the AS graph is the 'control plane' of Internet. The AS network of Internet has seen very rapid growth over the recent years (from about 3000 nodes in 1998 to about 25 000 nodes in 2008) and the growth of this network has been well documented with snapshots of the network being available on a regular basis [9]. As such, Internet AS networks present very realistic opportunities to gain insight into the evolution of complex networks.

The global structure of Internet AS networks is known to have a power law degree distribution (with scale-free exponent $\gamma = -2.2$ as reported in [10,11] for networks at that time) and a tier architecture. It is also known to display community structure and rich-club phenomenon [12]. The recently proposed measure of local assortativity [5] shows that Internet AS networks have a markedly different local assortativity profile to most other complex networks, including biological and social networks, which suggests that the evolution or growth of Internet is driven by fundamentally different design principles to that of other networks.

Several growth models exist to simulate the growth of the Internet, including the Inet 3.0 model [13], the Barabási-Albert model [14], the Generalized Linear Preference model [15], the Interactive Growth model [12], and the Positive Feedback Preference (PFP) model [16]. These models are capable of matching the degree distribution, and community structure (including the rich-club phenomena) of the Internet AS networks. However, these models do not generate topologies that match the local assortativity distribution of the Internet. In fact, as we will show, these models mostly generate local assortativity profiles that are much more similar to biological and social networks.

In this paper, we present a new growth model for Internet, called the Parallel Addition and Rewiring Growth (PARG) model. The PARG model satisfactorily explains the local assortativity distribution of the Internet, while retaining the ability to reflect the scale-free nature, and other properties explained by existing growth models. The PARG model we present rearranges links in parallel to addition of nodes, as is the case with real Internet growth. We make detailed comparisons between PARG model and other existing growth models. Finally, we outline possible applications and significance of the new growth model.

2 Local assortativity and its distribution

Assortativity is the tendency in networks where nodes mostly connect with similar nodes [3,17,18]. Similarity

[a] e-mail: mikhail.prokopenko@csiro.au

is typically defined in degrees. Naturally occurring networks display various levels of assortative or disassortative mixing. Formally, the assortativity r of a network is a correlation coefficient which is defined in terms of the networks degree distribution $p(k)$, its remaining degree distribution $q(k)$, and its link distribution $e_{j,k}$. The degree distribution $p(k)$ is a probabilistic distribution of encountering a node with a given number of links. The remaining degree distribution $q(k)$ is a related concept where the remaining degree means the number of 'remaining links' of a node that one sees when one travels along a randomly chosen link toward the node. It can be shown that compared to the degree distribution p, the remaining degree distribution q is biased toward higher degrees, and the relationship between the two distributions is given by

$$q(k) = \frac{(k+1)\,p(k+1)}{\sum_j jp(j)} \tag{1}$$

where the sum is over all nodes $1 \leq j \leq N$.

Given $q(k)$, one can introduce the quantity $e_{j,k}$ as the joint probability distribution of the remaining degrees of the two nodes at either end of a randomly chosen link. Given these quantities, network assortativity is defined [3,17,18] as a correlation function which is zero for nonassortative mixing, and positive or negative for assortative or disassortative mixing, respectively. That is

$$r = \frac{1}{\sigma_q^2}\left[\sum_{jk} jk\left(e_{j,k} - q(j)\,q(k)\right)\right] \tag{2}$$

where σ_q is the standard deviation of the remaining degree distribution of the network, $q(k)$.

Assortativity for a network is a single quantity which is measured at the global level, and it does not explain how individual nodes contribute to network assortativity, or what is the distribution of such contributions. Local assortativity (local assortativeness) has been recently proposed as a measure [5] which can be used to analyse assortative or disassortative tendencies at local level. Local assortativity, denoted by ρ, has been defined as an individual node's (direct) contribution to the network assortativity. It has been shown by Piraveenan et al. [5] that local assortativity of a node with degree $(j+1)$ is given by

$$\rho = \frac{(j+1)\left(j\overline{k} - \mu_q^2\right)}{2M\sigma_q^2} \tag{3}$$

where $\overline{k}$ is the average remaining degree of the node's neighbours, M is the number of links in the network, μ_q and σ_q are the mean and standard deviation of the remaining degree distribution of the network, $q(k)$, respectively. It follows that the network assortativity r is given by

$$r = \sum_{i=1}^{N} \rho_i \tag{4}$$

where N is the number of nodes in the network.

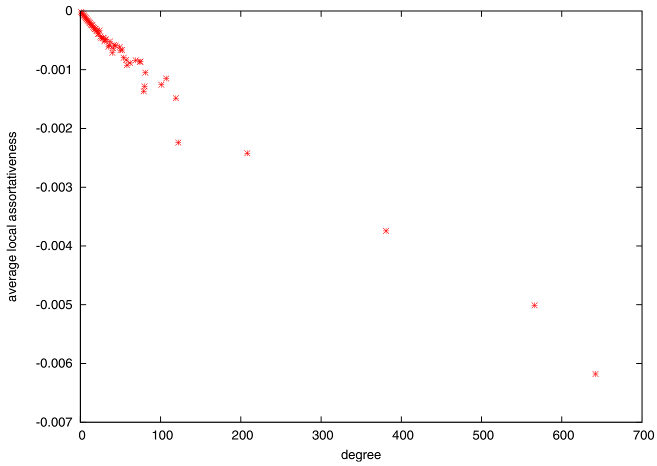

Fig. 1. (Color online) Local assortativity distribution of Internet at the AS level, in year 1998.

Since local assortativity is a property of a node, it is possible to construct local assortativity distributions for a given network, plotting local assortativity values against degrees. Since nodes with the same degree can have various local assortativity values, one can represent the average local assortativity value for all nodes with a given degree k, denoting this value as $\overline{\rho}(k)$. Then it follows from the previous equation that the network assortativity of a network can also be given by

$$r = N\sum_k p(k)\,\overline{\rho}(k) \tag{5}$$

where $p(k)$ represents the degree distribution of the network. The average local assortativity distribution, $\overline{\rho}(k)$ over degree k can now be constructed and analysed for complex networks.

3 Local assortativity distributions of Internet at the AS level

It has been observed that local assortativity distributions of Internet, unlike most of other scale-free networks, are roughly linear functions: that is, average local assortativity of nodes with degree k is roughly linearly and negatively correlated with the node degree [5]. Furthermore, all nodes in Internet topology are locally disassortative as shown in Figures 1, 2, 3.

It should be noted that most other real world networks display a different kind of local assortativity profiles. Specifically, the hubs in these networks are locally assortative, even if the network is overall disassortative. Some examples from biological and social networks are given in Figures 4, 5, 6, 7. It can be noted that all of these networks, as well as the Internet are scale-free networks and are globally disassortative (the network assortativity is a negative value, between $r = -0.1$ and $r = -0.4$).

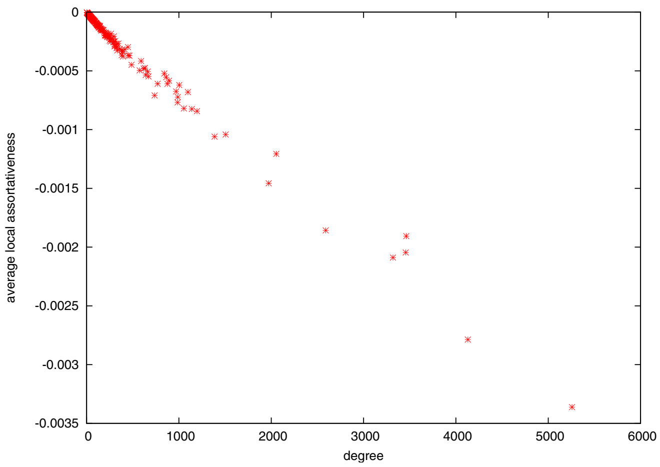

Fig. 2. (Color online) Local assortativity distribution of Internet at the AS level, in 2007, December.

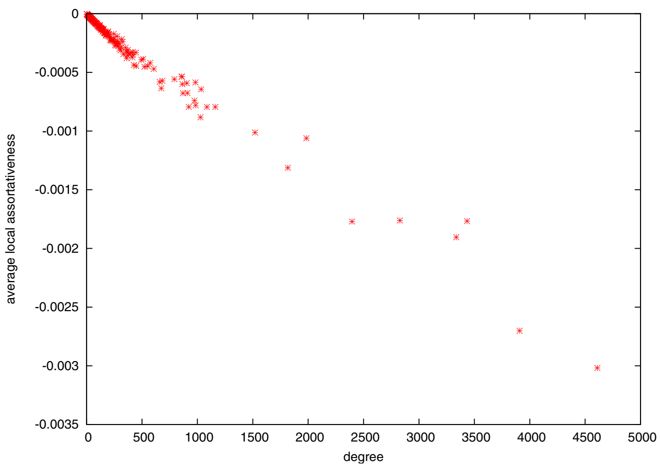

Fig. 3. (Color online) Local assortativity distribution of Internet at the AS level, in 2008, August.

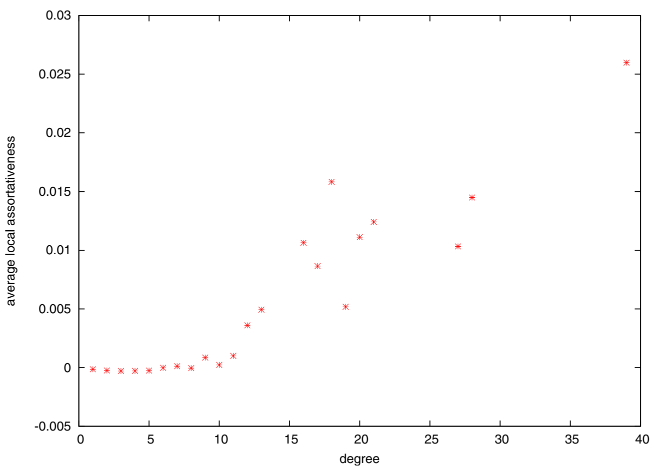

Fig. 4. (Color online) Local assortativity distribution of H. Sapien (human) Protein Protein Interaction network – data source: [19].

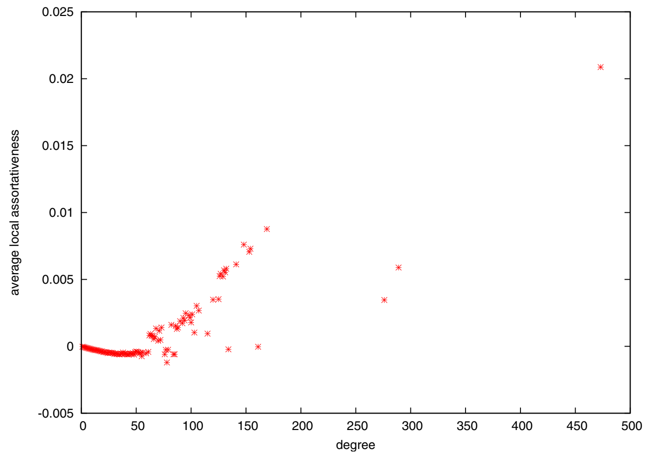

Fig. 5. (Color online) Local assortativity distribution of H. Sapien (human) Gene Regulatory network – data source: [20].

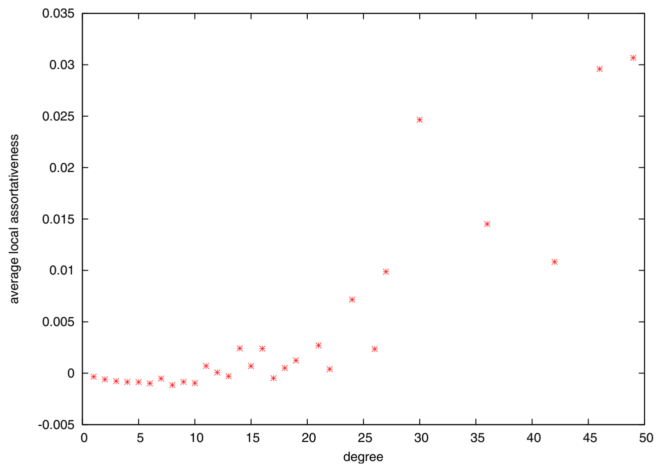

Fig. 6. (Color online) Local assortativity distribution of CA1 canonical signalling network (A canonical signalling network is not organism specific – data source: [21]).

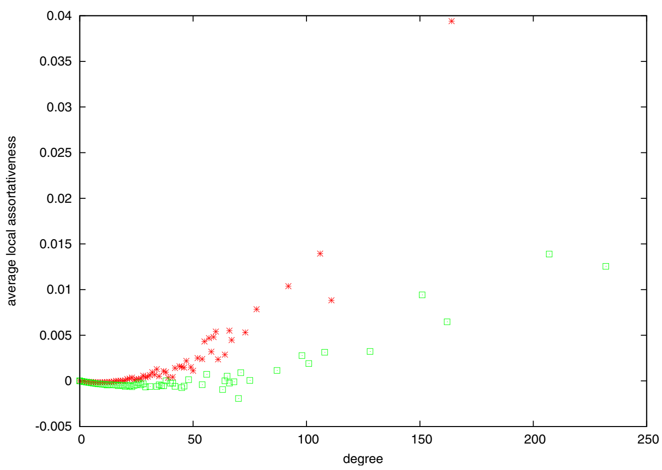

Fig. 7. (Color online) Local assortativity distribution of Citations networks: Scientometrics (stars), Small & Griffith and Descendants (squares). Reproduced from [5].

Therefore, we contend that the local assortativity distributions can be used to identify two classes of disassortative real world networks: (i) those that have assortative hubs and non-linear local assortativity profiles, and (ii) those that have disassortative hubs and linear local assortativity profiles. Most real world networks fall in the first category, whereas Internet seem to fall in the second category. This may mean that Internet growth is driven by fundamentally different design principles to other complex networks. This assertion brings us to the topic of growth models for Internet, which we can use to simulate the evolution of Internet over the years.

A number of growth models exist to simulate the growth of Internet [12], and most of these models are developed specifically to model Internet. Some of the most prominent models are Inet 3.0 model [13], the Barabási-Albert model [14], the Generalized Linear Preference model [15], a the Interactive Growth model [12], and the Positive Feedback Preference Model [16]. Let us briefly overview these existing models.

3.1 Inet 3.0 model

The Inet 3.0 model [13] is capable of matching the degree distributions of real AS Graphs. Given the degree distribution of the AS network that it needs to model, the Inet 3.0 mechanism assigns degrees to the given number of nodes to match the desired degree distribution, then connects these nodes using a three step process. Nodes are connected to other nodes with 'free' degrees using weighted linear preference. However, it has been noted that the model typically generates 25 percent less links than the real extended AS graphs [12].

3.2 The Barabási-Albert (BA) model

The BA model [14] first explained how a power law degree distribution can arise from a growth model, by introducing preferential attachment of nodes along with growth. In the BA model, new nodes attach themselves preferentially to nodes which already have a higher number of links. That is, the probability of an existing node i with degree k_i to be selected is

$$p_i = \frac{k_i}{\sum_j k_j}. \tag{6}$$

The BA model has inspired many growth models that followed and has been used as a starting point in some of them [12,15].

3.3 The Generalized Linear Preference (GLP) model

The GLP model [15] improves on the BA model by splitting the growth in two parts; (i) the addition of new nodes (ii) the addition of new links between existing nodes. starting with m_0 nodes connected by $m_0 - 1$ links, it performs one of the following operations at each time step. (i) with probability p, m new links are added between nodes chosen preferentially, and (ii) with probability $1-p$, one new node is added and connected to m existing nodes chosen preferentially. The probability of an existing node i with degree k_i to be selected is

$$p_i = \frac{k_i - \beta}{\sum_j k_j - \beta} \tag{7}$$

where β is a parameter, which when set to zero reduces the mechanism to exactly that of BA. Thus, it generalises the BA mechanism and matches the real AS graphs in degree distribution, clustering coefficient and path lengths.

3.4 The Interactive Growth (IG) model

The Interactive Growth model has been proposed recently [12] to model the rich club phenomena in the real AS graphs. The model starts with a random graph of m_0 nodes and the same number of links. At each time step (i) with 40 percent probability, a new node is connected to one host node and the host node is connected to two peer nodes (ii) with 60 percent probability, a new node is connected to two host nodes and one of the host nodes (randomly selected) is connected to one peer node[1]. Thus three new links are added at each time step.

The IG model thus determines the link density a priori, without parameterising it. However, it has been argued that the model captures the degree distribution and link distribution of the real AS graphs [12]. In addition, the model is able to capture the rich-club phenomena [22,23]. A rich-club is defined in terms of degree-based rank r of nodes, and the rich-club connectivity $\varphi(r)$. The degree-based rank denotes the rank of a given node when all nodes are ordered in terms of their degrees, highest first. This is then normalised by the total number of nodes. The rich-club connectivity is defined as the ratio of actual number of links over the maximum possible number of links between nodes with rank less than r. Thus, it is possible to calculate the rich-club connectivity distribution of a network, $\varphi(r)$ over r. It has been shown that the IG model is able to very closely match the $\varphi(r)$ over r distribution of real AS graphs [12].

3.5 The Positive Feedback Preference (PFP) model

The PFP model is derived from the IG model [16]. It follows a similar mechanism, except that to select the host and peer nodes it uses a non-linear preference called Positive Feedback Preference. In PFP, the selection probability

[1] There is no difference in 'type' between host nodes and peer nodes. A node which acted as a host node during one node addition could be selected as a peer node during another node addition. Any node to which the incoming node is directly connected is called the 'host' node, and any node then selected to make additional links with the host node is called the 'peer' node.

of a node is determined by

$$p_i = \frac{k_i^{1+\delta \log(k_i)}}{\sum_j k_j^{1+\delta \log(k_j)}} \quad (8)$$

where δ is a parameter of the model, e.g. $\delta = 0.021$. The non-linear preference is utilized to achieve better similarity to the AS graph's attributes compared to the IG model, for example in terms of getting the appropriate maximum degree.

The PFP model compares favourably with almost all measurable attributes of the AS graph, including the rich club coefficients [16].

4 Motivating the growth model: a network motif with linear local assortativity distribution

The models reviewed in the previous section have been shown to model the degree distribution and community structure (in terms of rich-club connectivity) of the Internet reasonably well, as well as reasonably account for Internet's scale-free nature. However, these models have not been validated for Internet's local assortativity distributions.

Therefore, we used each of these models to grow networks and calculated the local assortativity distributions. We observed that these models fail to capture the local assortativity distribution of Internet. Specifically, these growth models (except the PFP model to some extent), along with the Erdös–Rényi random network model [1,2,24], display a local assortativity profile where the hubs are assortative, as Figures 8, 9, 13, 14 show. Such models each have a number of parameters, and we verified that changing these parameters do not affect the overall local assortativity profile. For example, Figure 13 show the Barabási-Albert model with the expected number of links per node (E_{link}) changed. The network assortativity changes with this parameter; however, the local assortativity profile remains non-linear and the hubs remain assortative.

The PFP model to some extent is an exception, and we studied it in detail as it has been shown to be the best model around to model AS networks by some distance. The local assortativity profiles generated by PFP model are shown in Figures 10, 11, 12. As these figures shows, the PFP model is capable of producing local assortativity profiles with disassortative hubs, though it does so for values of δ higher than the recommended $\delta = 0.021$. In any case, even for higher values of δ, the profile is not perfectly linear or as evenly spread out as that of the AS networks is. Specifically for higher values of δ which seem necessary to produce disassortative hubs, the profile seems punctuated, i.e there is a big 'gap' between the degrees of the biggest hub and other networks. This is not surprising given the nature of the positive feedback mechanism of the model, in which 'the rich not only get richer, but get disproportionally richer', as the authors of the model

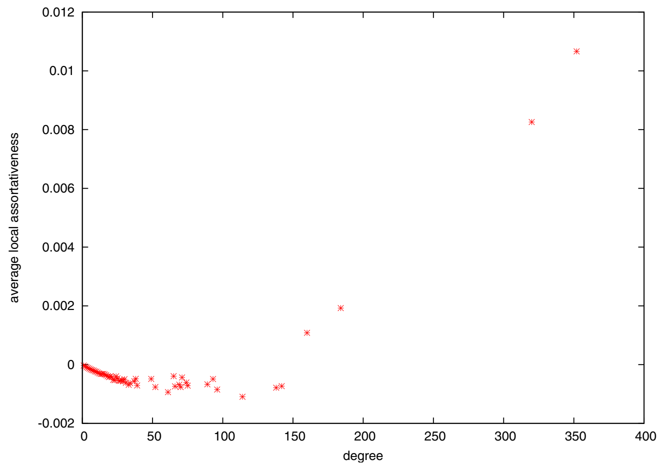

Fig. 8. (Color online) Local assortativity distribution of a network grown with preferential attachment (The Barabási-Albert Model). The network size is that of Internet AS network in 1998.

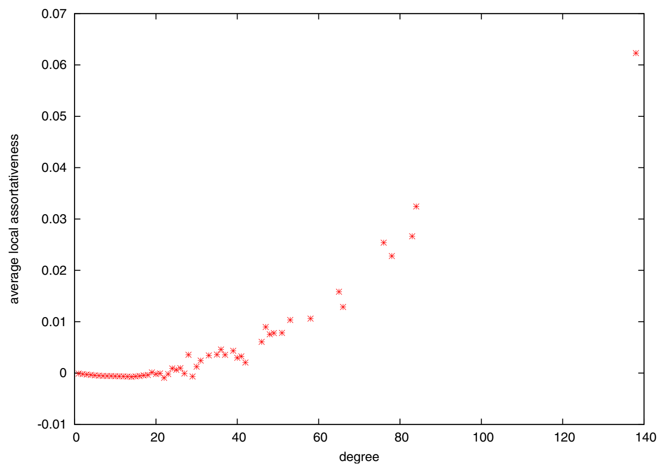

Fig. 9. (Color online) Local assortativity distribution of a network grown using the Interactive Growth model proposed by [12]. The network size is that of Internet AS network in 1998.

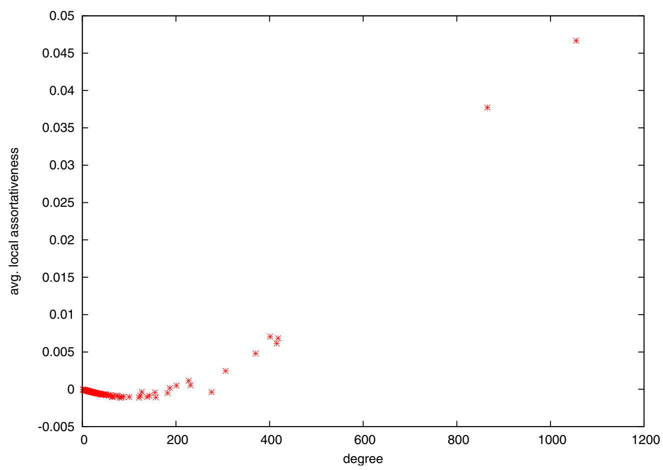

Fig. 10. (Color online) Local assortativity profile of a network grown using the PFP model for $\delta = 0.011$.

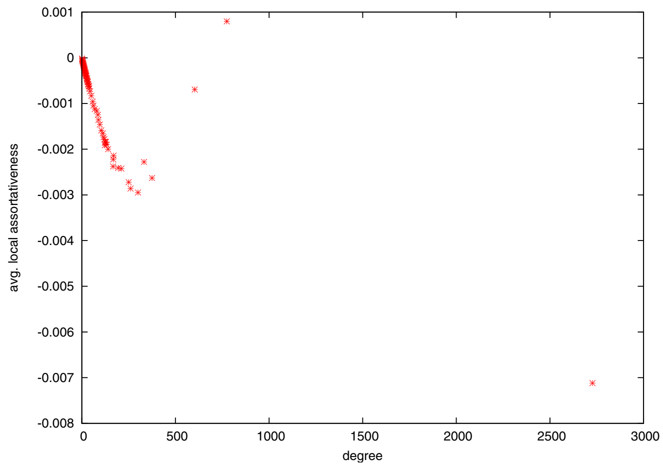

Fig. 11. (Color online) Local assortativity profile of a network grown using the PFP model for $\delta = 0.021$.

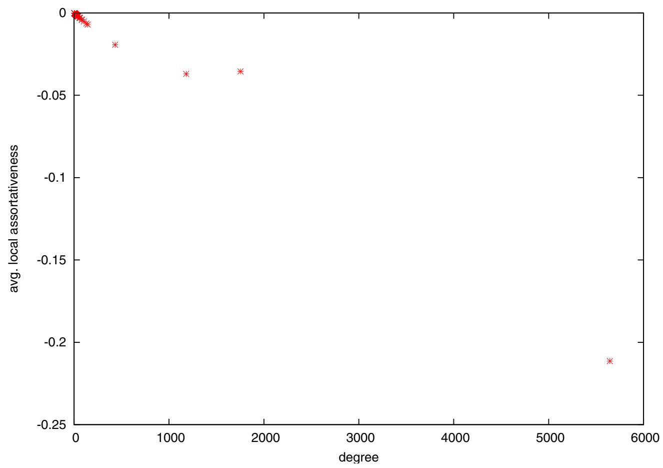

Fig. 12. (Color online) Local assortativity profile of a network grown using the PFP model for $\delta = 0.042$.

point out [16], and that effect is even more pronounced for high δ, which implies strong positive feedback. Therefore, while the PFP model overall is quite effective in modelling the AS networks, it produces a punctuated linear local assortativity profile.

Figures 1, 2, 3, 8, 9, 13, 14, 10, 11, 12 illustrate that the existing models produce non-linear (or punctuated) local assortativity profiles, warranting a new growth model. Such a new model should capture not only the local assortativity distribution of Internet, but also the attributes already captured by the existing models – namely degree distributions and community structure. In this paper we present such a growth model, which we call the *Parallel Addition and Rewiring Growth* (PARG) model. The model is presented in the following section. Before presenting a step-by-step description of the model, we explain the motivation behind it.

To motivate the new growth model, it is important first to recognise a network motif which has the property of perfect linear local assortativity distribution, as well as being scale-free. Then a network can be constructed

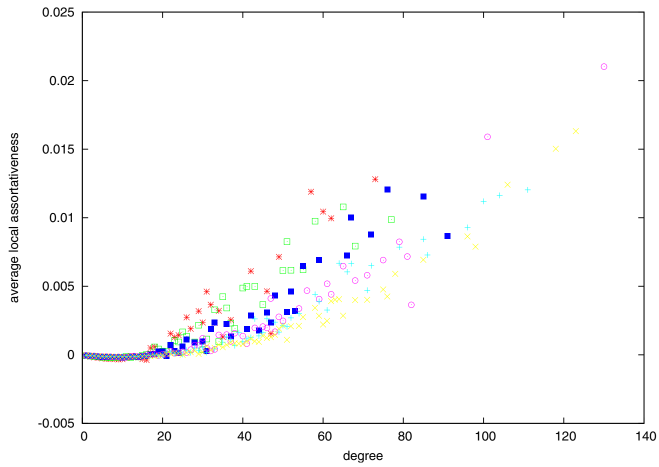

Fig. 13. (Color online) Local assortativity distribution of networks grown with the Barabási-Albert model, with various parameters $E_{link} = 0.5, 0.75, 1.0, 1.25, 1.5$ and 2.0. Note that the general shape of local assortativity profile remains non linear with assortative hubs.

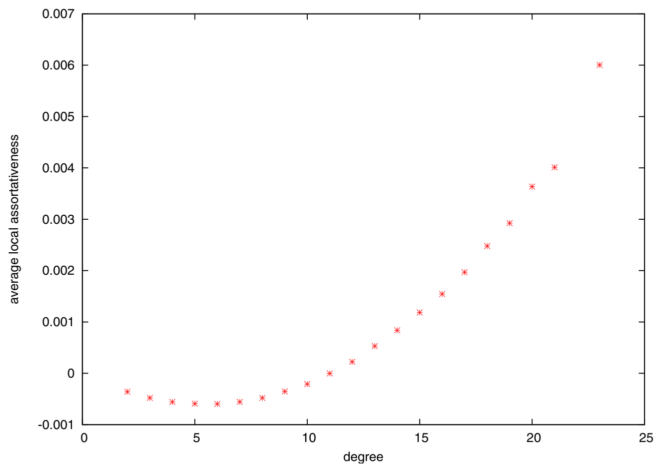

Fig. 14. (Color online) Local assortativity distribution of a network grown as an Erdös-Rényi random graph. The network size is that of Internet AS network in 1998.

by connecting together such motifs or growing them in a scale-free manner.

Let us first note that a star motif has this property at a very elementary level [5]. A star motif can have nodes with only two different degrees, and all nodes are locally disassortative - the local assortativity distribution of a star motif is shown in Figure 15 [5].

Clearly, if we consider a number of larger star motifs with varying maximum degrees as a network, the local assortativity distribution of the overall network would be negatively linear. However, these star motifs have to be interconnected to form a single network, without compromising the locally disassortative nature of the nodes. This can be accomplished if the hubs in the stars are connected to peripheral nodes in other stars, so that the links are disassortative in nature. The likelihood of hub-to-hub links

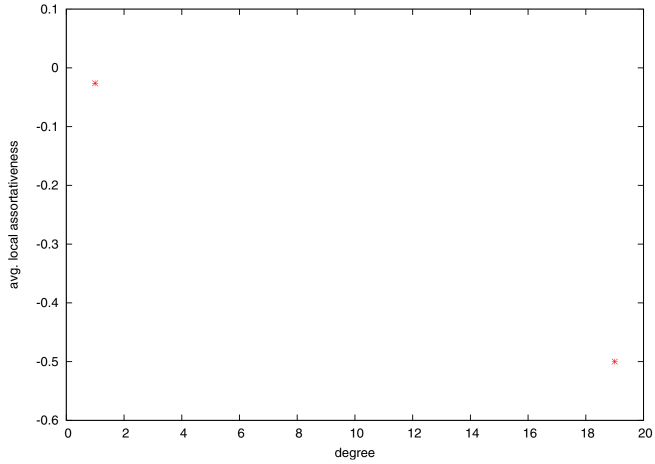

Fig. 15. (Color online) Local assortativity distribution of a star motif with highest degree = 19.

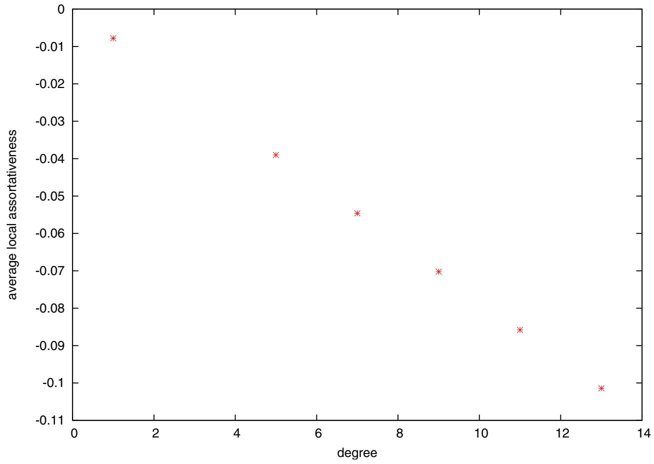

Fig. 17. (Color online) The local assortativity distribution of the network motif displayed in Figure 16.

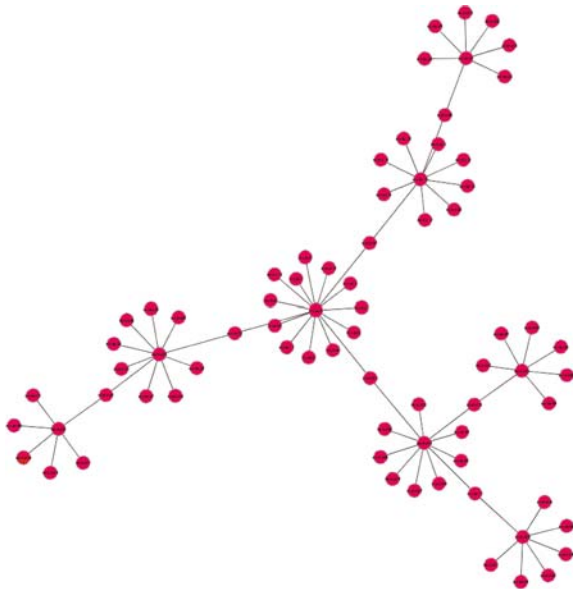

Fig. 16. (Color online) A network motif that displays negative linear local assortativity distribution. Note that this is essentially a hierarchy of stars with hubs connected together via linking nodes.

must be reduced, so that they only form a very small proportion of all links in the network. Such a network is shown in Figure 16. Note that this network is scale-free.

It is evident that a growth model that satisfies the local assortativity distribution of the Internet has to contain many instances of the pattern above. However, when the network is sufficiently large, a number of assortative links can also appear, as they are not likely to affect the distribution in a big way. Indeed, Internet AS networks are by no means a hierarchy of stars as the above motif is. On the other hand, AS networks have been shown to display the 'rich-club' phenomena, where most of the hubs are densely connected to each other [12,16]. Nevertheless, many such hierarchies of stars must be interwoven in the Internet topology for the overall network to maintain a linear local assortativity profile.

We also made a few other observations about the topology of Internet in designing our growth model. We noted that nodes are constantly being deleted as well as added in the Internet topology. For example, between January 2004 and February 2004, 2431 new nodes were added while 2061 were deleted, making a net increase of 370 nodes [9]. This however, is not mainly due to Internet Service providers going out of business, but due to the permanent variation of interconnections [11]. Connections between AS members are constantly rearranged and may flicker, and if a AS node has only a few connections, it is actually possible that all connections may be shut down from time to time [11]. At such times, it appears as if the node has been deleted. The actual node mortality is small compared to this. Therefore, it is important that node deletion, as well as addition, needs to be explicitly modelled. None of the existing growth models take this into account. While the network is growing on average and as such can be modelled by purely joining nodes, the connection patterns are affected by the deletion. For example, the preferential attachment model assumes that all joining nodes preferentially attach themselves to existing nodes. However, those nodes that are deleted are not likely to be chosen in a preferential way; therefore the resulting patterns in the link distribution are not wholly captured.

Taking these facts into account, we present a growth model that resembles the preferential attachment mechanism only in part. That is, nodes join preferentially with existing hubs, making the formation of giant hubs possible. At the same time, another mechanism is at work, which disfavours assortative links. That is, assortative links are replaced by disassortative links, giving way to the emergence of star-like motifs and ultimately a linear local assortativity profile.

Let us note here, however, that the growth model we present here is a generic model to generate linear disassortative local assortativity profiles. While it has been motivated by the Internet, it is not a model limited to Internet

Table 1. Parameters of PARG model and rich-club phenomena based on AS 98 networks. N is the network size.

PARG			Top 1% rich club	Top 2% rich club	Maximum Degree
N_{add}	N_{del}	N_{cut}			
0.04	4.8	$1 + 0.04N$	27%	11%	650
0.006	4.8	$1 + 0.2N$	35%	18%	655
AS 98			37%	17%	641

growth. However, with a suitable set of parameters it captures reasonably well other features of Internet AS networks such as degree distribution, maximum degree and rich club coefficients.

We do not explicitly model deleting nodes in PARG model, though we do explicitly model deleting links. Service agreements between Internet service providers (ISP) appear and disappear all the time, and as such the disappearance of links have to be modelled explicitly. An ISP going out of business would be comparably less frequent. Furthermore, the constant deletion of links in parallel to the growth of network seems to be one of the driving forces behind the linear local assortativity profile and disassortative hubs that the AS network displays. Now we proceed to present a step-by-step description of the PARG model.

5 The PARG model for Internet growth

The PARG model contains two mechanisms of growth. One is the node-addition mechanism, and the second is the link rearrangement mechanism. These mechanisms work in parallel. Specifically, each time a node is added to the network some links are rearranged stochastically. The rate of re arrangement is a parameter of the model.

Our node-addition mechanism closely reflects the BA model of Internet growth [14], as it has been shown that this model sufficiently explains the scale-free nature and power law degree distributions of Internet. Below we present the model in detail.

The model starts the growth from a small initial network of size N_0. The initial network could be a simple random graph.

At each time step a new node is added to the network.

- The new node stochastically makes N_{add} number of links with existing nodes. That is, the joining node makes a number of links with the expected number of links being N_{add}.

The new node connects to the existing nodes preferentially. That is, a node's probability to be selected to have a link with the joining node is proportional to the number of its existing links. Formally, the probability of an existing node i with degree k_i to be selected is

$$p_i = \frac{k_i}{\sum_j k_j}. \quad (9)$$

After each node addition:

- Probabilistically choose and delete N_{del} number of assortative links in the network. This is done in the following fashion:
  - choose N_{cut} number of the highest degreed nodes from the network (sort nodes based on degree and choose the first N_{cut} number of nodes);
  - each link in the selected node is stochastically deleted with a probability that is inversely proportional to the degree of that node;
  - the actual probability is calculated so that the expected number of link deletions is maintained at N_{del}.

Formally, if a node with degree d from the network has degree-based rank $rank_d \geq N_{cut}$, the probability of a link of that node being deleted is:

$$p = \frac{N_{del}}{d\ d_{cut}} \quad (10)$$

where d_{cut} corresponds to the degree of the node that has exactly the rank of $rank_d$. Otherwise (if $rank_d < N_{cut}$) the probability of a link of that node being deleted is:

$$p = 0. \quad (11)$$

- Delete the chosen links.
- For each deleted link, add *two* links to the network;
  - the node with the higher degree among the two nodes that were connected by the deleted link, node s, is chosen as the node to create these new links from;
  - another two nodes p_1, p_2 are selected from the network;
  - these nodes are selected in anti-preferential fashion. That is, nodes in the network are sorted according to degree, the highest degreed first. The probability of a node being selected is proportional to its *rank* in the sorted list. The higher the rank, the higher the probability is to get selected, e.g., a node that is ranked 20th is twice as likely to be selected than a node that is ranked 10th;
  - two new links are created: one connecting s with p_1, and another connecting s with p_2.

This process is repeated until the desired number of nodes are added.

N_{add}, N_{del} and N_{cut} are parameters of our model. The values of these parameters that were used in our simulation experiments are summarised in Table 1. The parameters N_{add} and N_{del} together determine the number of links in the network.

The idea behind the second mechanism is that assortative links are deleted and replaced by disassortative links. Note that we are selecting links from high-degreed nodes to be deleted, so these links are likely to be assortative.

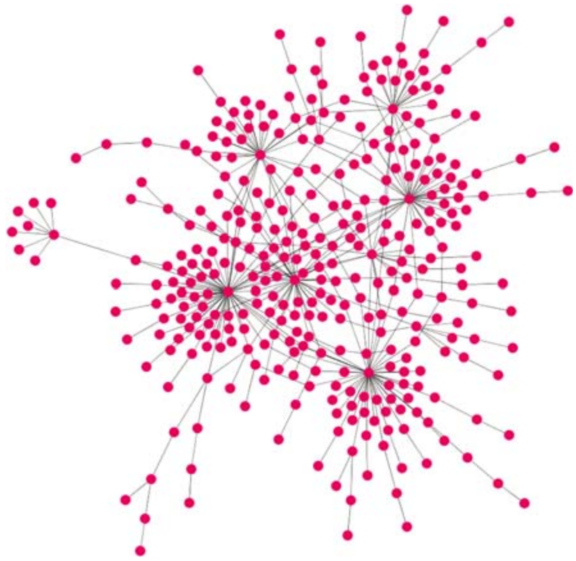

Fig. 18. (Color online) A sub-network of 350 nodes grown by the PARG model. The original grown network contained 3000 nodes, out of which 350 nodes were randomly chosen with their links to illustrate the connection patterns. Note that it is highly similar, albeit bigger, to the motif we proposed earlier.

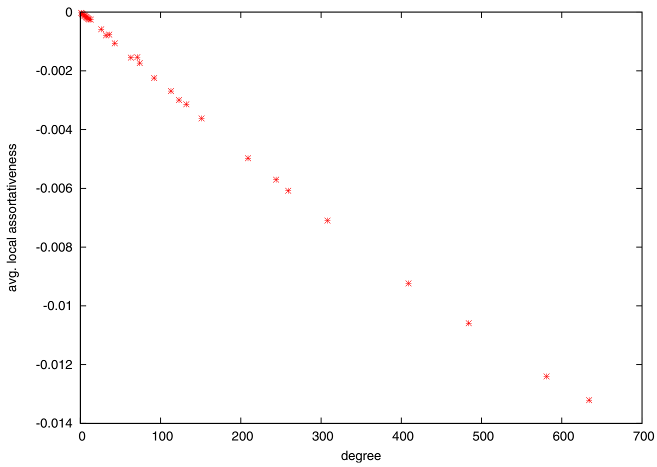

Fig. 19. (Color online) The local assortativity distribution of a network grown by the PARG model. Network size = 3000 nodes. Number of Links = 6100. Note that these values correspond roughly to the size of Internet AS network in 1998.

When the links are replaced, though, the node with the higher degree (s) is chosen for one end of these links – but the nodes at the other end (both p_1 and p_2) are chosen anti-preferentially. This effectively results in disassortative links. The deletion/replacement of such links counters, to some extent, the nature of preferential association where hubs are preferred to form links. The end result is that while hubs are allowed to form, they are discouraged to form links with other hubs excessively. The hubs among AS network connect mostly to relatively peripheral nodes, and while they maintain links with other hubs, such links are only a small proportion of all the links the hubs may have.

Let us note also that the PARG model growth will result in some nodes being 'dropped' from the network, even though such occurrences will be rare. Specifically, when a link is deleted and replaced by two links to the node which has the higher degree, the node with lower degree may drop out of the network if it had only that link which has been deleted. As we explained above, this however is also the case with real AS networks, where nodes do drop out temporarily during network growth. This is yet another aspect of Internet growth that the PARG model captures.

Figure 18 shows a sub-network of 350 nodes grown by the PARG network. It is possible to observe that the network contains many star motifs and the hubs are often not directly connected, though some assortative links are visible too. A visual comparison with the Internet AS networks is not possible since these are much larger and the design patterns cannot be clearly discerned from a figure. However, as we show in the next section, the PARG model seems to capture well the topological design patterns of Internet, including the local assortativity distribution.

In the next section, we analyse the performance of the PARG model.

6 The local assortativity distribution of networks grown by the PARG model

Figure 19 shows the local assortativity distribution of a network produced by the PARG model. It can be noted that the PARG model produces a local assortativity distribution which is linear and negative with disassortative hubs – similar to the real Internet AS network. As Figures 20, 21 show, the networks produced by PARG model are also scale-free and their degree distributions are compatible with the degree distributions of the Internet. Table 1 shows that the PARG model can produce rich-club coefficients and maximum degrees comparable with AS graphs. Thus, the PARG model is successful in producing the desired local assortativity profile while retaining other aspects of Internet that are already modelled.

We undertook a detailed comparison study of the PARG model with other existing growth models. For this purpose we considered the Barabási-Albert (BA) model, Interactive Growth (IG) model, Generalized Linear Preference (GLP) model, Positive Feedback Preference (PFP) model, as well as our PARG model, contrasted with the real AS networks. Using PARG model, we have grown networks compatible with both AS 98 (3000 nodes) and AS 2008 (25 000 nodes) networks. We considered the power law exponent γ and assortativity r of the grown networks, as well as the ability to produce scale-free characteristics, the nature of hubs, the linearity of local assortativity profiles, and the ability to produce community structure. Our results are summarised in Table 2.

Table 2 clearly shows that only the PARG model is able to produce a network that has a linear local assortativity profile and disassortative hubs, as is the case with

Table 2. A comparison between growth models and the real Internet AS network.

Network	AS 1998	AS2003	AS2008	PARG	PARG	BA	IG	GLP	PFP	Inet
Size	3000	16 000	25 000	3000	25 000	3000	3000	3000	3000	3000
Scale-free	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Power law exponent γ	–2.22	–2.16	–2.04	–2.08	–1.96	–2.9	–2.1	–2.1	–2.2	–2.1
assortativity r	–0.198	–0.14	–0.13	–0.28	–0.26	–0.09	–0.28	–0.2	–0.23	–0.2
Assortative hubs	No	No	No	No	No	Yes	Yes	Yes	No	Yes
Linear local assortativity	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes/No	No
Rich club	Yes	Yes	Yes	Yes	Yes	No	Yes	No	Yes	No

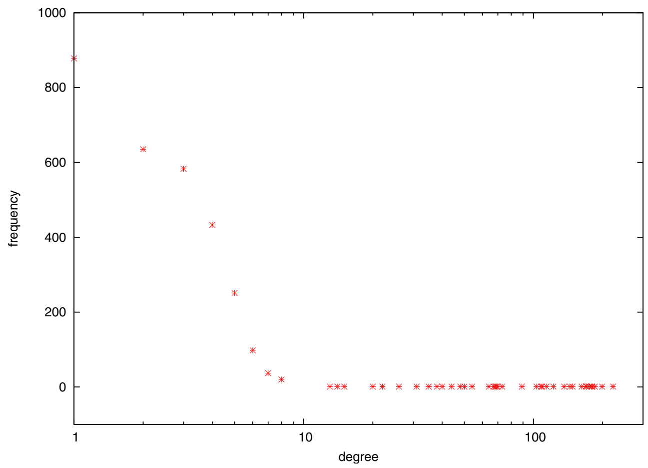

Fig. 20. (Color online) Degree distribution of a network grown by the PARG model. Nodes = 3000. Links = 6700 (roughly corresponds to AS 98 network). Network assortativity $r = -0.28$.

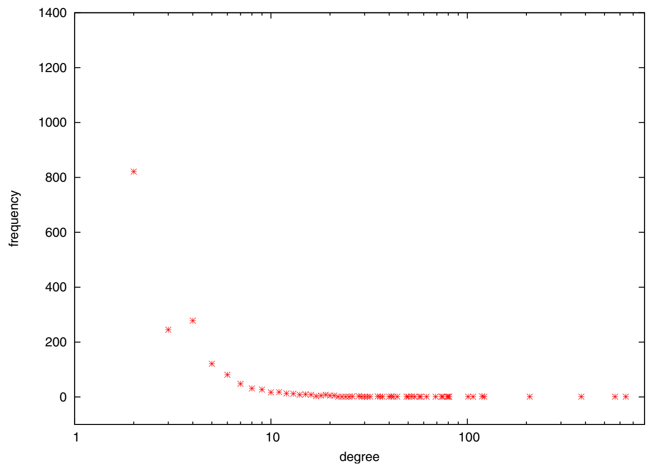

Fig. 21. (Color online) Degree distribution of the real AS 98 network. Nodes = 3000. Links = 6100. Network assortativity $r = -0.198$.

real AS networks. Meanwhile, the PARG model reasonably retains the ability to model other aspects of Internet topology, such as degree distribution and community structure.

It is pertinent here to discuss in some detail the capacity of the PARG model to produce the rich-club phenomena. The PARG model has been motivated by a desire to grow networks which show linear local assortativity profiles with disassortative hubs. However, having disassortative hubs does not mean that the hubs cannot be interconnected. It merely means that a very high proportion of the links of these hubs are connected to comparatively peripheral nodes. It should be noted that, as pointed out in Piraveenan et al. [5], the rich-club phenomenon is in some way connected to local assortativity, albeit that the rich-club phenomenon is concerned with hubs only and cannot be used to analyse peripheral nodes alone. In other words, the rich-club coefficient represents the cumulative local assortativity aggregated from the highest-degreed nodes toward the smaller degreed nodes [5]. The relationship between the rich-club coefficient $\varphi(r)$ and the local assortativity $\rho(r)$, both plotted over rank r, has been explored in [5]. On the other hand, networks which show nonlinear local assortative profiles with assortative hubs (class I), and networks which show linear local assortative profiles with disassortative hubs (class II) may both display strong rich club connectivity. As such, rich club connectivity and local assortativity remain related but independently relevant concepts to analyze complex networks. The PARG model is able to capture both in a way comparable to real AS graphs. The one-percent and two-percent rich club connectivities of the PARG model is shown in Table 1, along with the maximum degrees of the produced networks.

7 Conclusions

In this paper, we have presented a new growth model – the PARG model – which grows networks with a linear local assortativity profiles and disassortative hubs. It is a dynamic model that includes two parallel mechanisms: a node addition mechanism which is similar to the preferential attachment, as well as a link rearrangement mechanism which ensures a linear local assortativity distribution for the network. The growth model satisfies the recently introduced local assortativity distribution for real AS networks. The model also captures link deletion and nodes dropping out (mostly temporarily) as a result, which occurs in real AS networks but has not been hitherto captured by existing growth models.

We have compared the PARG model with existing growth models for Internet. We found that the PARG model captures the degree distribution and rich-club phenomena as well as do other existing models, in addition to being unique in capturing local assortativity profiles. We observed that the PFP model produces severely punctuated and not perfectly linear local assortative profiles. We have highlighted that naturally occurring disassortative networks can be classified into two classes based on local assortativity profiles, namely (i) those with assortative hubs and non-linear local assortativity distributions, and (ii) those with disassortative hubs and linear local assortativity distributions. The PARG model, though motivated as a growth model to explain local assortativity profiles of the Internet, could be used to model any network that belongs to the second class. Therefore, together with the newly proposed measure of local assortativity, this growth model has the potential to greatly aid the simulation, design and analysis of complex networks in general.

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