Network structures from optimization principles

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By discovering that considerable insights about many real-world systems can be gained by characterizing them as networks, network science has attracted considerable attention over the last decade. Seminal studies have revealed seemingly universal classes of networks, of which scale-free and small world networks are the most prominent examples. While the first are characterized by degree distributions that obey power laws with typical exponents in the range between -2 and -3, the latter have topologies that combine the local cohesiveness of spatial grids with the logarithmic scaling of average path lengths with system size typical for random graphs. Scale-free and small world networks describe a large number of important real-world systems, ranging from social networks to networks in biology and ecology to engineered systems such as the internet. A focus of the work in the field has been to understand mechanisms that explain the universal structure of the interaction topologies in these systems.

Two principal approaches have contributed to understanding network structures so far. The first are assembly mechanisms, that derive the structure of large complex networks from processes that describe the piecewise addition of nodes and links according to (often stochastic) rules over time. Preferential attachment is an important mechanism in this category. The second approach is via optimization, thus assuming that a network structure observed in the real-world represents the end point of some guided reorganization mechanism that aims at optimizing system performance during its evolution.

In my talk I will make use of the latter approach and discuss the explanatory power of optimization principles for small world and scale free network structures. In the focus of the first part of the talk will be the guided evolution of networks subject to spatial constraints. Attempting to explain the formation of small worlds, I will investigate the constrained evolution of networks towards path length- and synchrony optimality. Spatial constraints are included via a fitness function that combines the desired goal (i.e. the propensity of a network to synchronize or the average shortest path length of the network) with a requirement for shortest length of wire that would be needed to connect the nodes in space. The larger the influence of the cost of wire, the more severe the influence of spatial constraints.

Varying constraints two novel types of small worlds are discussed: small worlds with power law distributed link length and highly modular small worlds. The first arise when node locations are fixed in space and a spatially constrained network configuration that optimizes synchronization or path lengths is thought. The second are found, when node locations are free to change while path length or synchronization properties of the networks are optimized.

The example of path length optimal spatially constrained networks is used to provide some analytical guidance for the simulation results. On the basis of this example I link power laws in the link length distribution to hierarchical network organizations and describe how different types of hierarchical organization translate into different exponents of the power law decay of the link length distribution.

(If time permits) I will use the second part of the talk to discuss the formation of scale free networks from optimization principles. For this, I again make use of the example of distance optimizing networks, but this time I consider the interplay of a process of network assembly with a network optimization process that operates at a different timescale. Networks are optimized by guided rewiring towards path length optimality for a time T_{opt}, after which new nodes are introduced and randomly connected to existing nodes. Exploring the phase diagram of evolved network structures depending in the ratio of optimization- and assembly timescales, scale free networks are found in a large parameter range in which neither the optimization nor the assembly processes are dominant.

Related work:

1. M. Brede "Synchrony-optimized networks of non-identical Kuramoto Oscillators", Phys. Lett. A 372 2618-2622 (2008).
2. M. Brede "Local vs. global synchronization in networks of non-identical Kuramoto oscillators", Eur. Phys. J. B 62, 87-94 (2008).
3. M. Brede, “Construction Principles for Highly Synchronizable Sparse Directed Networks”, Phys. Lett. A 372, 5305-5308 (2008)
4. M. Brede, “Synchronization on directed small worlds: feed forward loops and cycles”, Europhysics Letters 84, 40004 (2008).
5. M. Brede and B. de Vries, “Networks that optimize a trade-off between efficiency and dynamical resilience”, Physics Letters A 373(43), 3910-3914 (2009).
6. M. Brede, “Optimal synchronization on strongly connected directed networks”, Eur. Phys. J. B 74, 217-225 (2010).
7. M. Brede, “Optimal synchronization in space”, Phys. Rev. E 81, 025202R (2010).
8. M. Brede, “Coordinated and uncoordinated optimization of networks”, submitted to Physical Review E (2010)
9. M. Brede, “Small worlds in space: Synchronization, spatial and relational modularity”, submitted to Europhysics Letters (2010)
10. M. Brede, “Growth and optimality in network formation”, submitted to Physical Review E (2010).