Assortativity in cyber-physical networks

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Introduction

Many complex systems are amenable to be described as networks (Solé and Valverde, 2004; Albert and Barabasi, 2002; Albert et al., 1999; Newman, 2003), and it has been a recent trend to study common topological features of such networks. Network diameter, clustering coefficients, modularity, community structure, information content are some features analysed in recent literature (Alon, 2007; Lizier et al., 2009; Piraveenan et al., 2009a; Prokopenko et al., 2009). One such measure that has been analysed extensively is assortativity (Newman, 2002; Albert and Barabasi, 2002; Newman, 2003; Callaway et al., 2001). Having originated in ecological and epidemiological literature (Albert and Barabasi, 2002), the term 'assortativity' refers to the correlation between the properties of adjacent network nodes. Based on degree-degree correlations, assortativity has been defined as a correlation function: the networks that have a positive correlation coefficient are called assortative; while the networks characterised by a negative correlation coefficient are called disassortative.

The precise local contribution of each node to the global level of assortative mixing — "local assortativity" — can also be quantified (Piraveenan et al., 2008, 2009b, 2010). Local assortativity profiles (as distributions of local assortativity over nodes' degrees) may be constructed for various networks, and used to classify cyber-physical networks (Piraveenan et al., 2008).

In this paper, our objective is to characterise classes of networks in terms of the unbiased formulation of local assortativity (Piraveenan et al., 2010).

Definitions and Terminology

Consider a network with N nodes and M links. Assortativity for such a network has been defined as a correlation function (Newman, 2002), in terms of the network's excess degree distribution $q(k)$, and link distribution $e_{j,k}$. The excess degree is the number of remaining links encountered when one reaches a node by traversing a link. The link distribution of the network is the joint probability distribution of the excess degrees of the two nodes at either end of a randomly chosen link. The formal definition of network assortativity is given by:

$$r = \frac{1}{\sigma_q^2}\left[\left(\sum_{jk} jk e_{j,k}\right) - \mu_q^2\right] \tag{1}$$

where $e_{j,k}$ is the link distribution of the network, σ_q is the standard deviation of the excess degree distribution of the network, $q(k)$, and μ_q is the expectation of the distribution

Local assortativity was motivated in Piraveenan et al. (2008) by calculating the contribution of each node to the above correlation coefficient. The unbiased representation of local assortativity (Piraveenan et al., 2010) is given by

$$\hat{\rho}_v = \frac{j\,(j+1)\left(\overline{k} - \mu_q\right)}{2M\sigma_q^2} \tag{2}$$

where j is the excess degree of node v; $\overline{k}$ is the average excess degree of its neighbours, and $\sigma_q \neq 0$. The sign of the local assortativity (positive or negative) is determined by the difference between the average excess degree ($\overline{k}$) of the neighbours and the global average excess degree (μ_q). If the neighbours' average is higher, then the node is assortative. If the global average is higher, the node is disassortative. Therefore, the local assortativity can also be defined as a scaled difference between the average excess degree of the node's neighbours and the global average excess degree (the scale factor is proportional to the product of the node's degree and excess degree).

We will utilise average local assortativity plotted against degree. Average local assortativity $\overline{\rho}(d)$ can be calculated by averaging local assortativity quantities of all nodes with a given degree d. Local assortativity is a quantity that involves both degree and average (neighbour) degree, and as a result, the local assortativity profiles clearly differ from average degree profiles. In particular, an average degree profile always contains positive values that increase with the degree, while local assortativity profiles may contain both positive or negative values, increasing or decreasing with the degree.

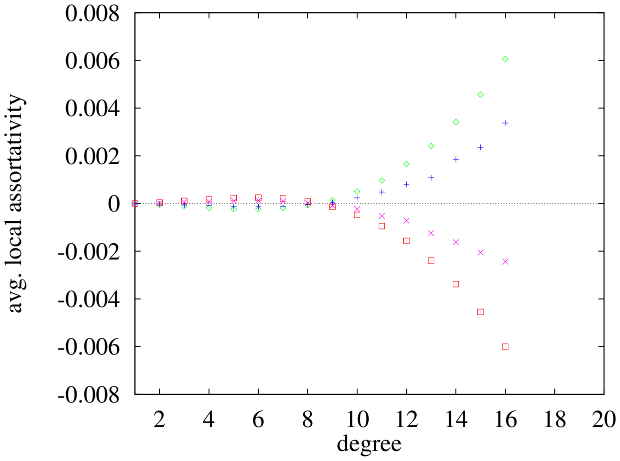

Figure 1: Local assortativity profile of scale-free networks ($N = 1000$ and $\gamma = 2.1$) with $r = 1.0$ ('◊'), $r = 0.5$ ('+') , $r = -0.5$ ('×') and $r = -1.0$ ('□').

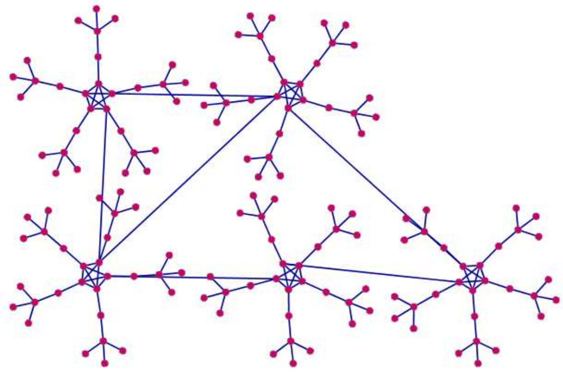

Figure 2: Example of a disassortative network with assortative hubs — class (iii), $r = -0.109$.

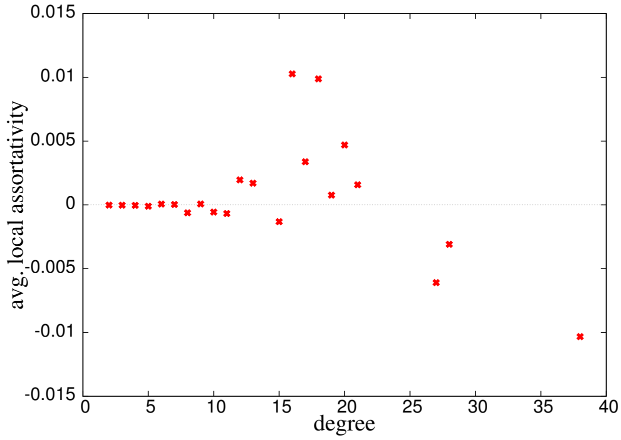

Figure 3: Local assortativity profile of H. sapiens Protein-Protein Interaction network — class (iv); $r = 0.075$.

Classification of networks

We begin the analysis by constructing model Barabási–Albert scale-free networks (Albert and Barabasi, 2002) of various assortativity levels and observing their local assortativity profiles. Some of the results are shown in Figure 1 for network size $N = 1000$ and power law exponent $\gamma = 2.1$.

We could observe from Figure 1 that the profiles are symmetric with respect to the degree axis when assortativity is varied from $r = 1.0$ to $r = -1.0$ while other network parameters are kept constant. We also note that (i) globally assortative networks have assortative hubs and disassortative low-degree nodes (many real-world metabolic networks are in this class), and (ii) globally disassortative networks have disassortative hubs and assortative low-degree nodes (typically, food-webs are in this class). That is, the overall assortativity of the network is matched by that of the hubs.

However, a model network given in Figure 2, with the overall assortativity $r = -0.109$ has assortative hubs. This example represents a third class, demonstrating that it is possible to have (iii) disassortative networks with assortative hubs. On the other hand, some assortative networks exhibit disassortative hubs, such as the Protein-Protein Interaction (PPI) network of H. sapiens (Figure 3. A number of other PPI networks displayed a similar profile. These networks represent the fourth class, namely (iv) the assortative networks with disassortative hubs.

The disassortative tendency of highly interacting nodes to be connected to low-interacting ones in PPI networks is quite informative in the context of cyber-physical systems — it was argued that "this effect decreases the likelihood of cross talk between different functional modules of the cell and increases the overall robustness of a network by localizing effects of deleterious perturbations" (Maslov and Sneppen, 2002).

Conclusions

Analyzing a range of model and real-world networks, we observed four classes of networks, namely: (i) assortative networks with assortative hubs, (ii) assortative networks with disassortative hubs, (iii) disassortative networks with disassortative hubs, and (iv) disassortative networks with assortative hubs. Real-world examples for the first three classes were identified, and a model network was constructed as an example for the fourth class.

The local assortativity profiles provide a quantitative tool for analysis and design of cyber-physical networks. These profiles highlight important topological differences in otherwise seemingly indistinguishable networks. This may help in studying diverse network properties and dynamics: e.g., (a) network growth may be modelled in such a way that the grown networks not only satisfy global characteristics, but also agree with required local assortativity profiles (Piraveenan et al., 2009b); (b) network robustness may be analysed in terms of an attack targeting the nodes with higher local assortativity; (c) motifs within networks can be studied via their average local assortativity, etc.

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